Learning shallow quantum circuits

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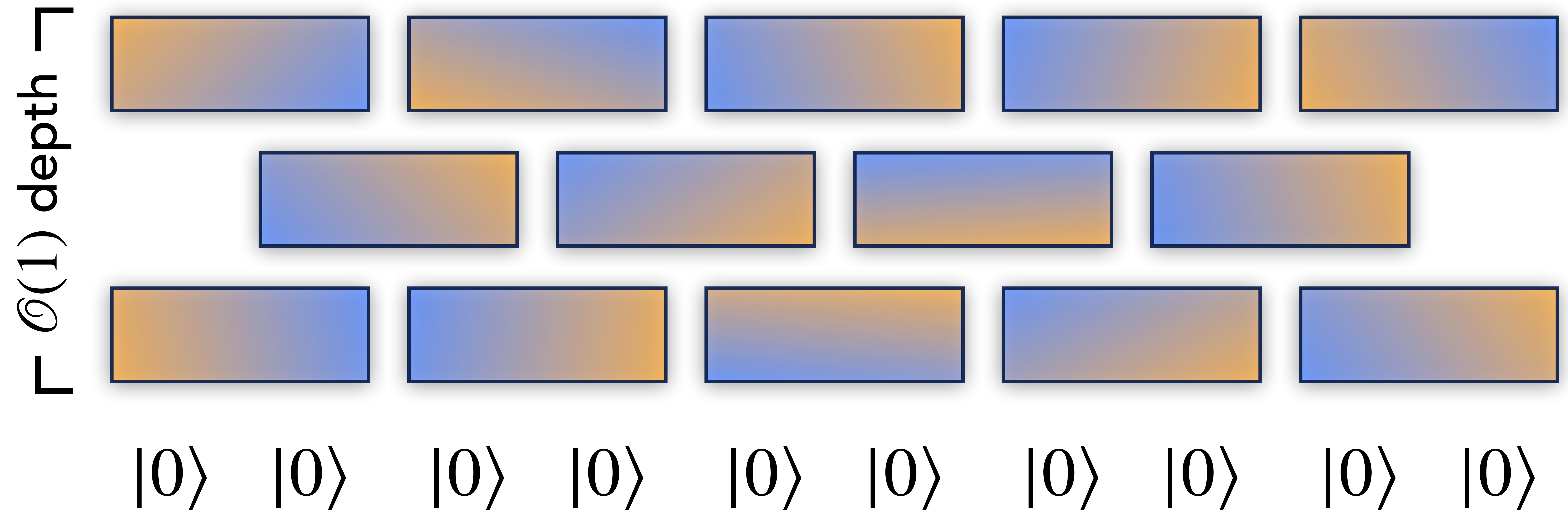
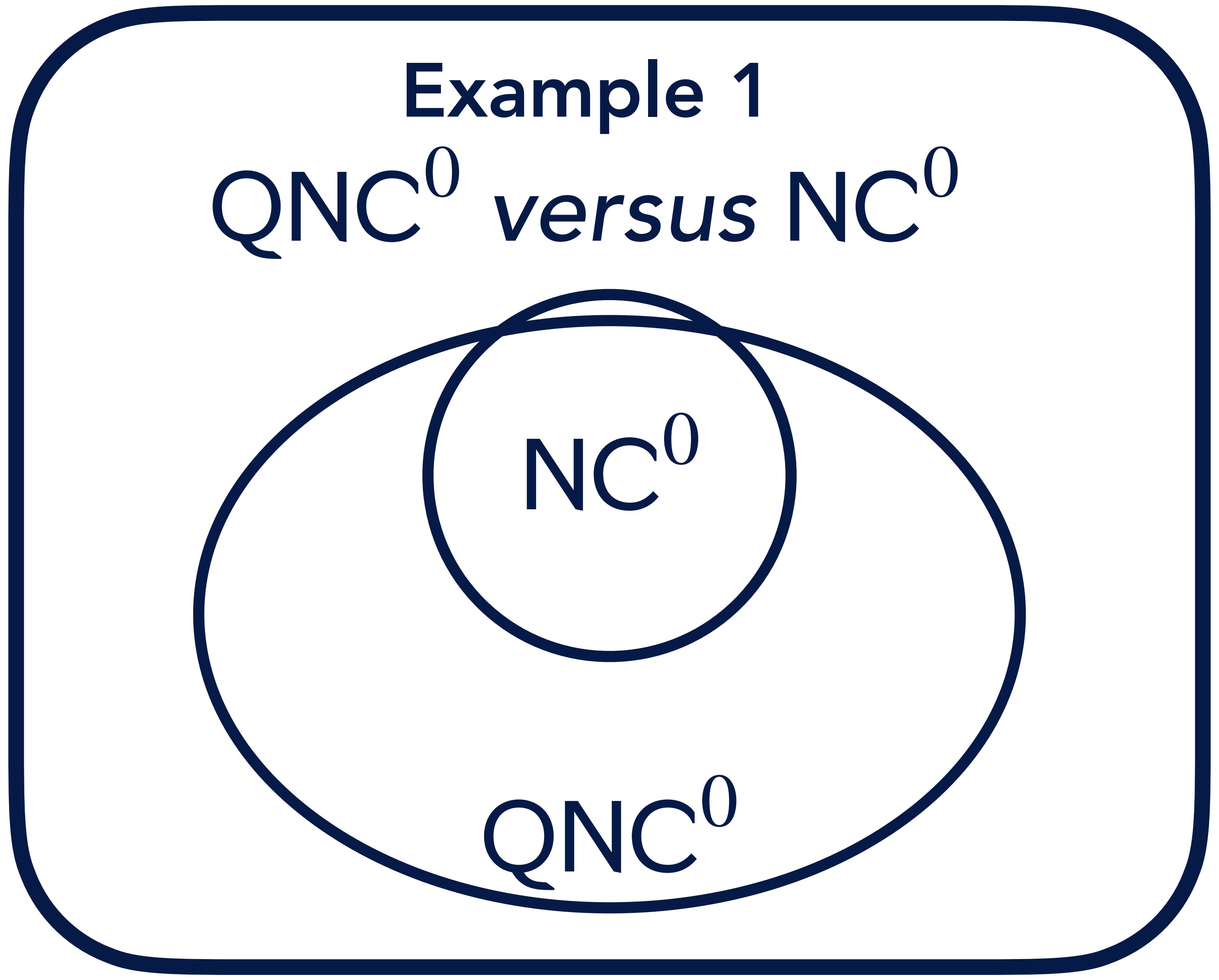
Shallow quantum circuits

- Shallow quantum circuits are quantum circuits with constant depth.
- Despite their simplicity, they are surprisingly **powerful**.



Shallow quantum circuits

- **Example 1:** Shallow quantum circuits are **unconditionally more powerful** than shallow classical circuits.



Quantum advantage with shallow circuits. Science 2018

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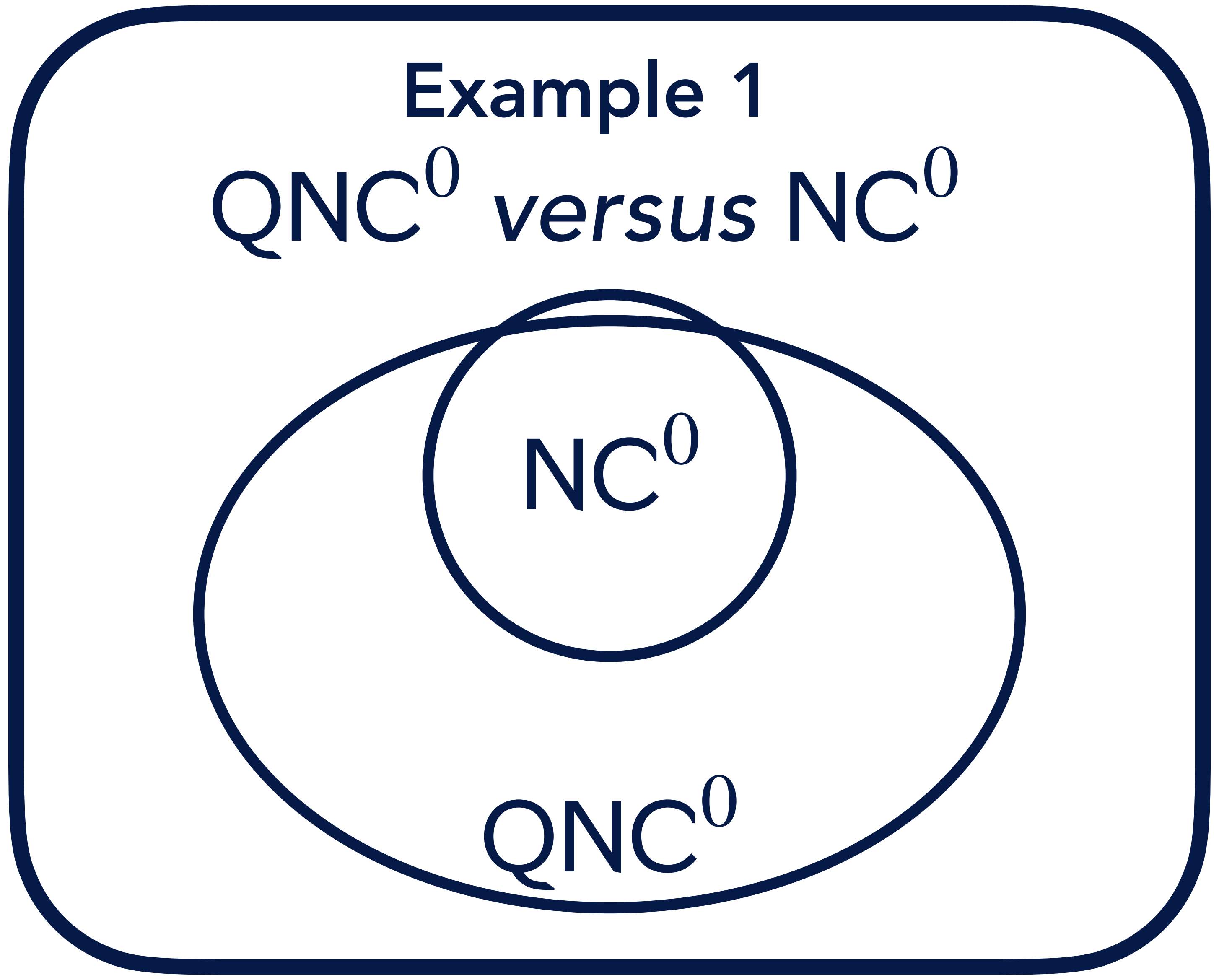

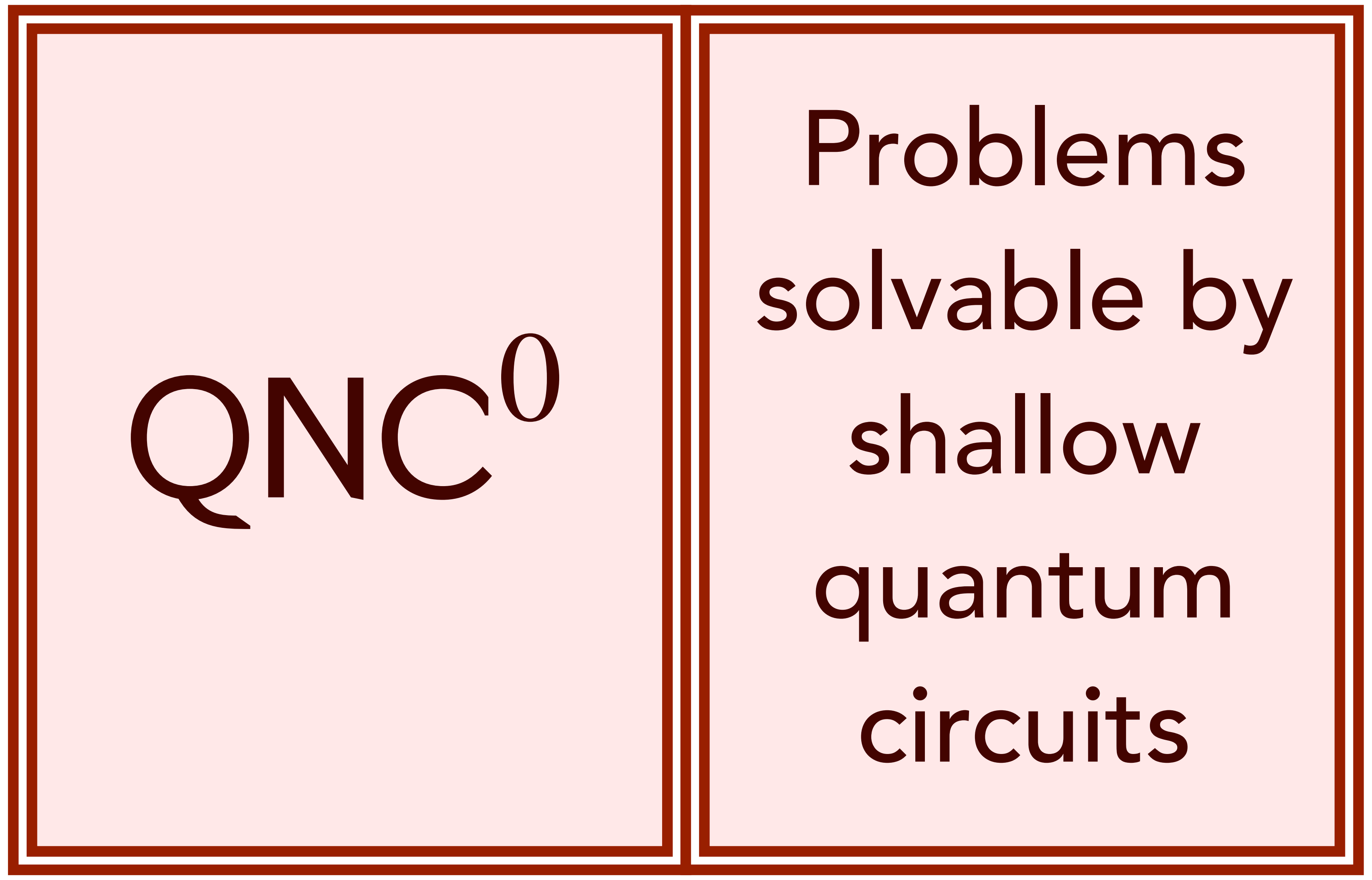

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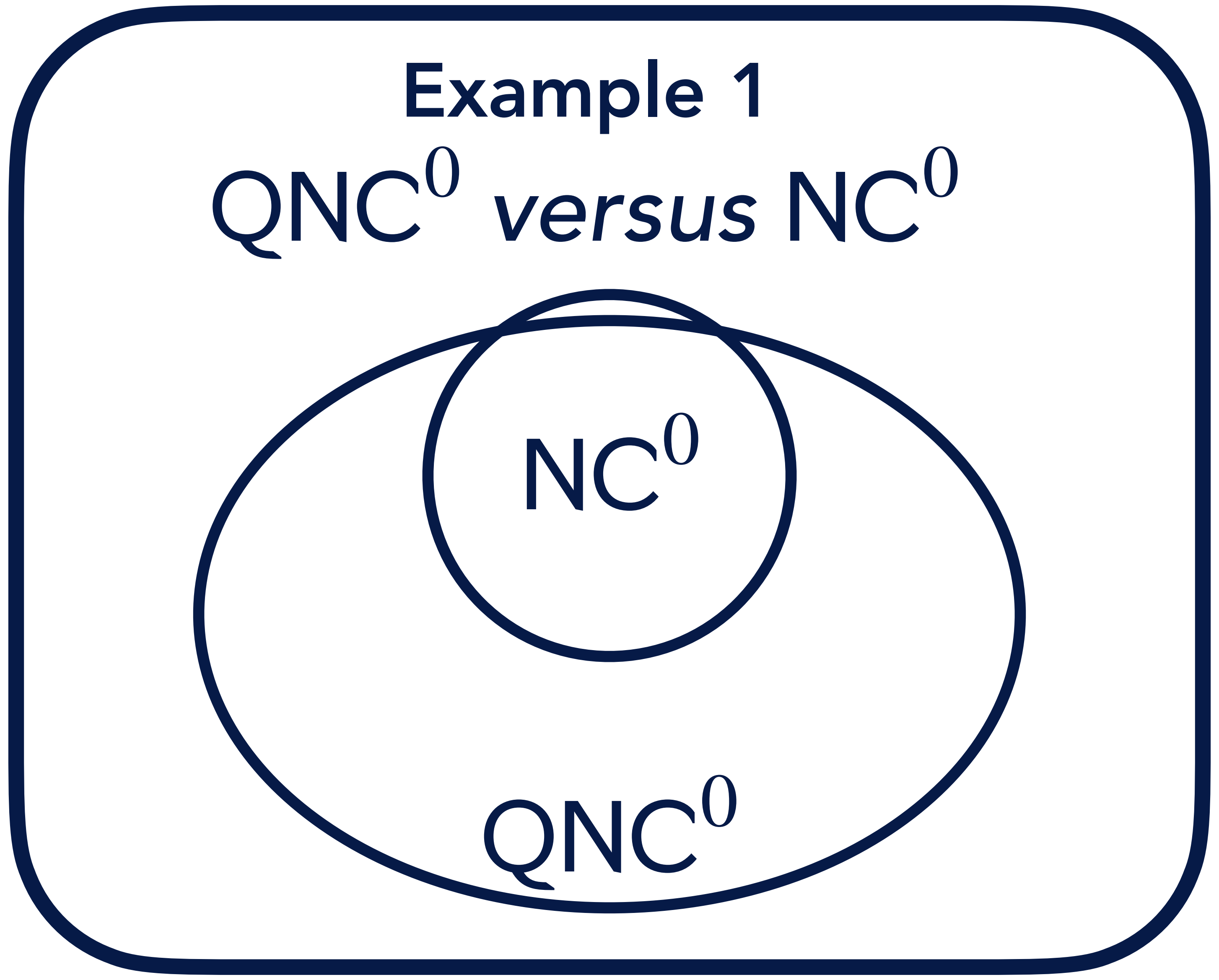

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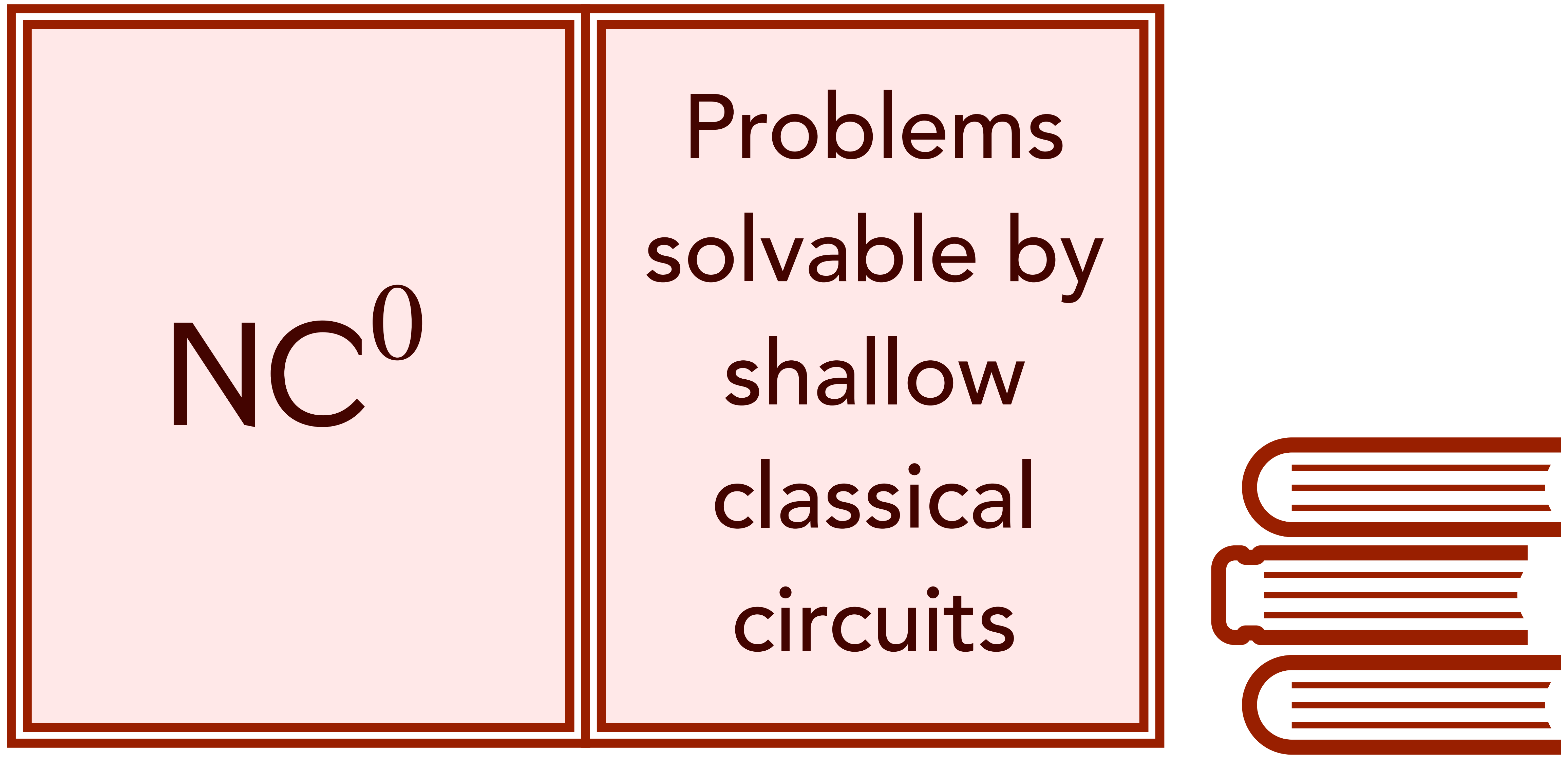

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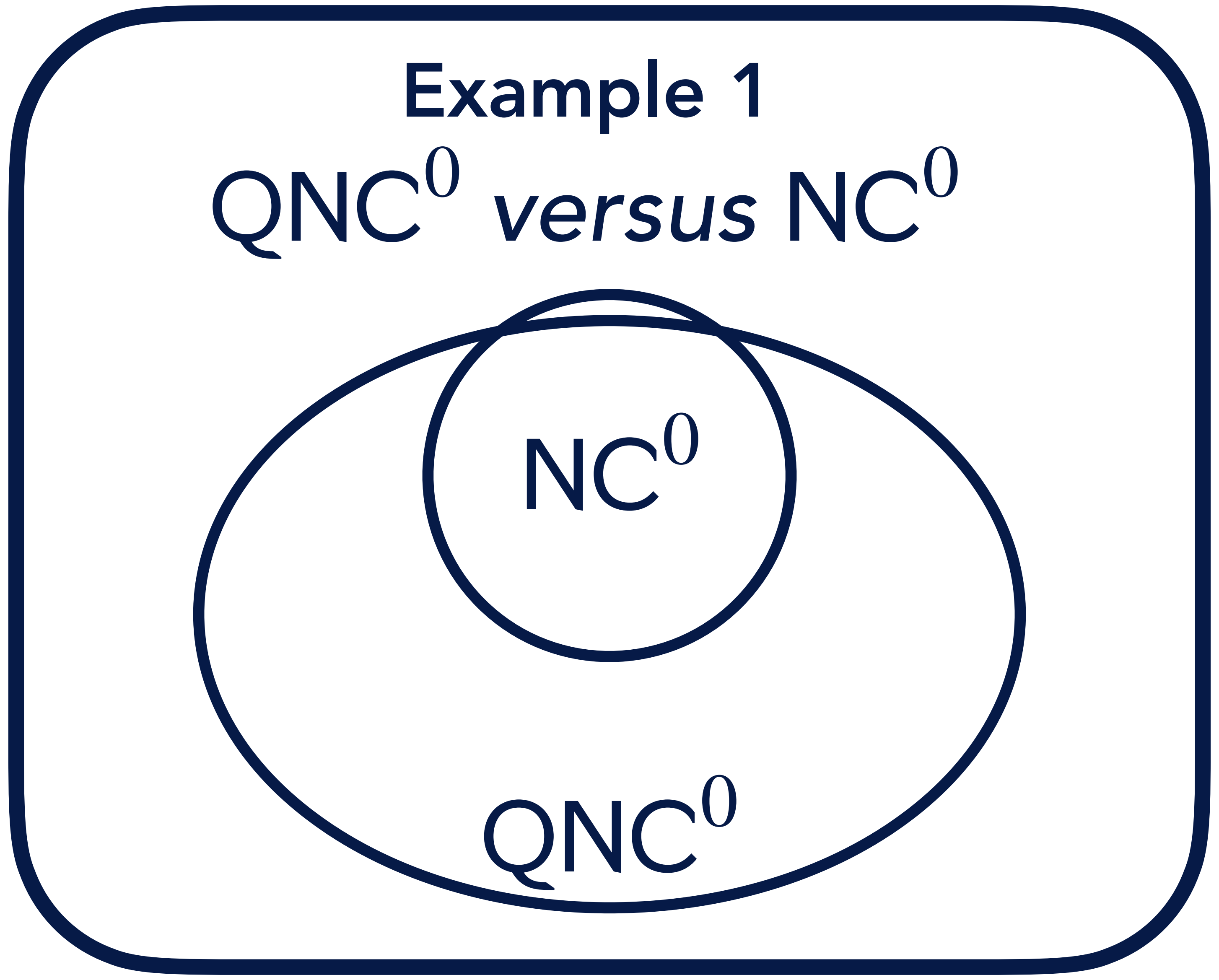

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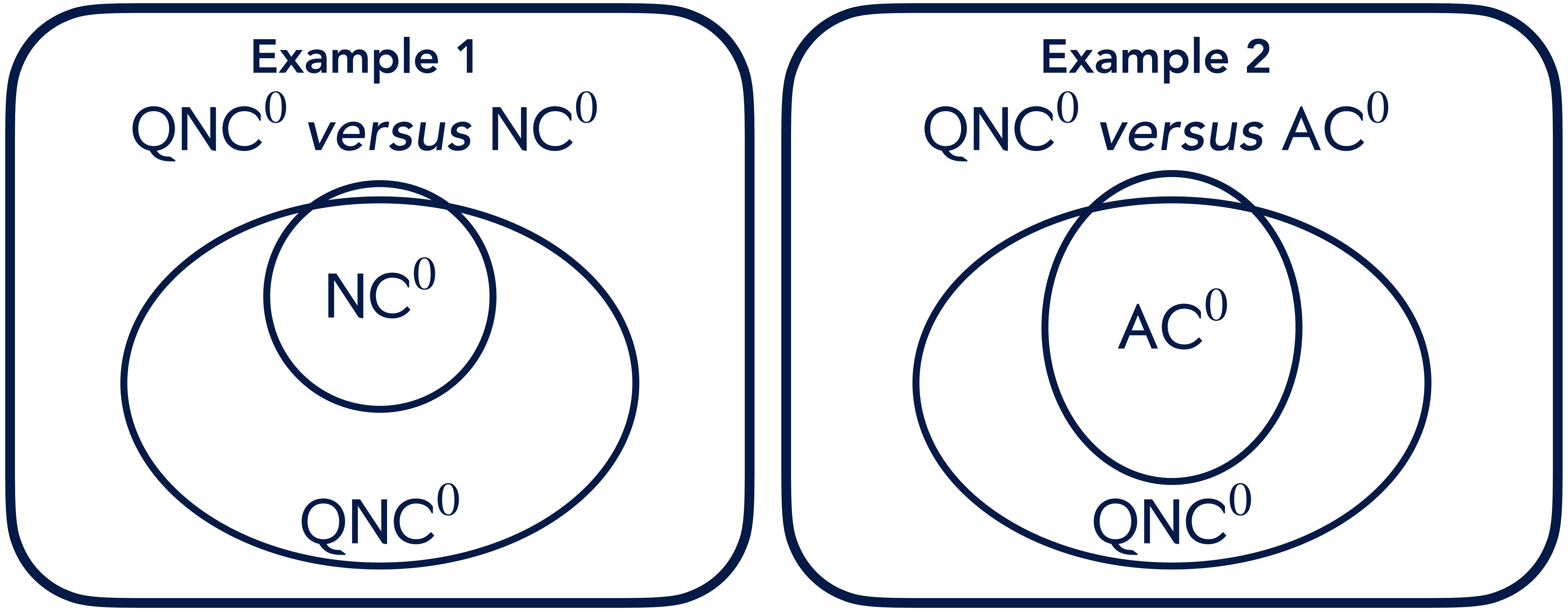
Shallow quantum circuits

- **Example 2:** Shallow quantum circuits are **expon. more powerful** than shallow classical circuits with unbounded fan-in AND gates.



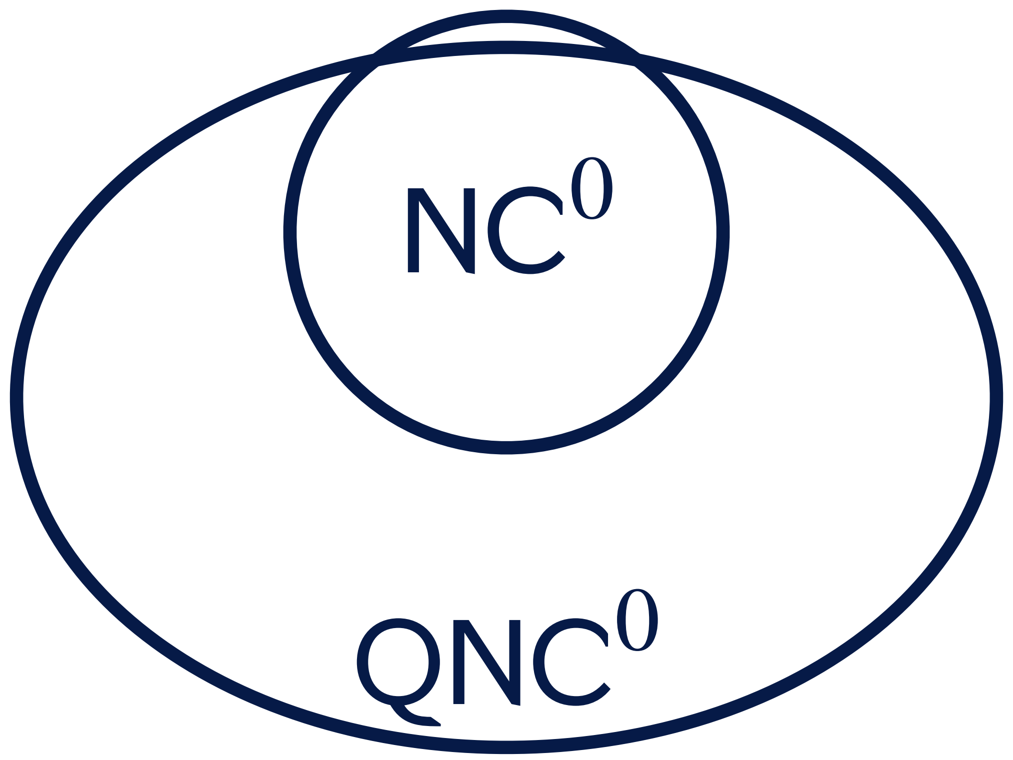
Exponential separation between shallow quantum circuits and unbounded fan-in shallow classical circuits. STOC 2019

Shallow quantum circuits

- **Example 2:** Shallow quantum circuits are **expon. more powerful** than shallow classical circuits with unbounded fan-in AND gates.

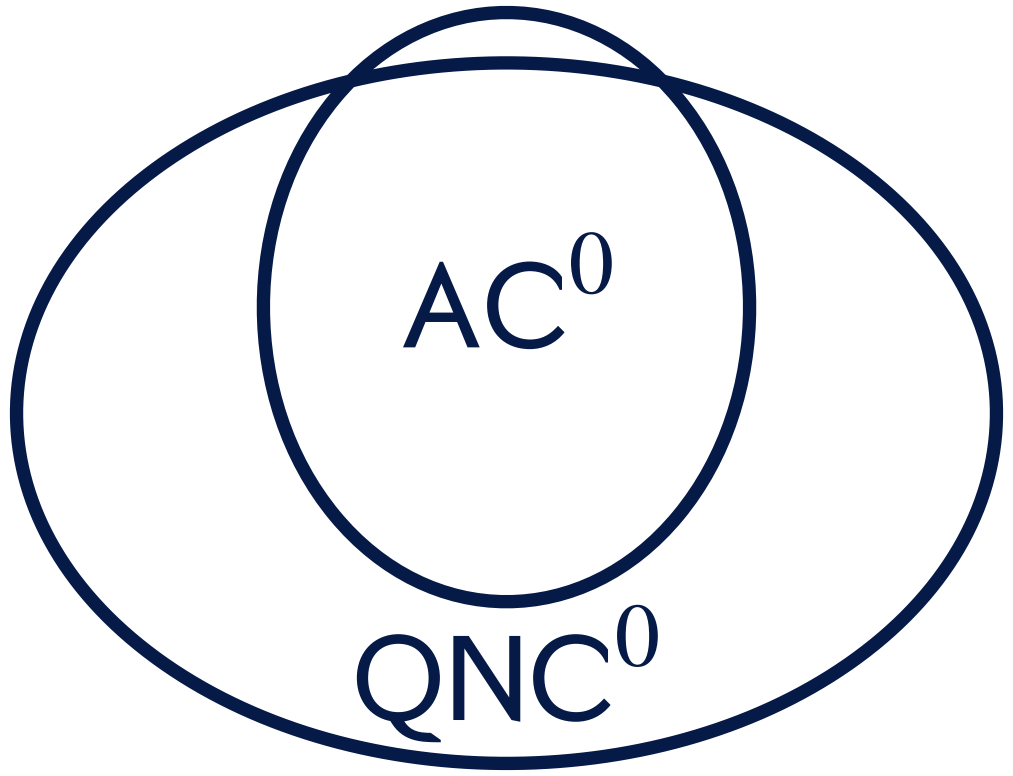
Example 1

QNC^0 *versus* NC^0



Example 2

QNC^0 *versus* AC^0

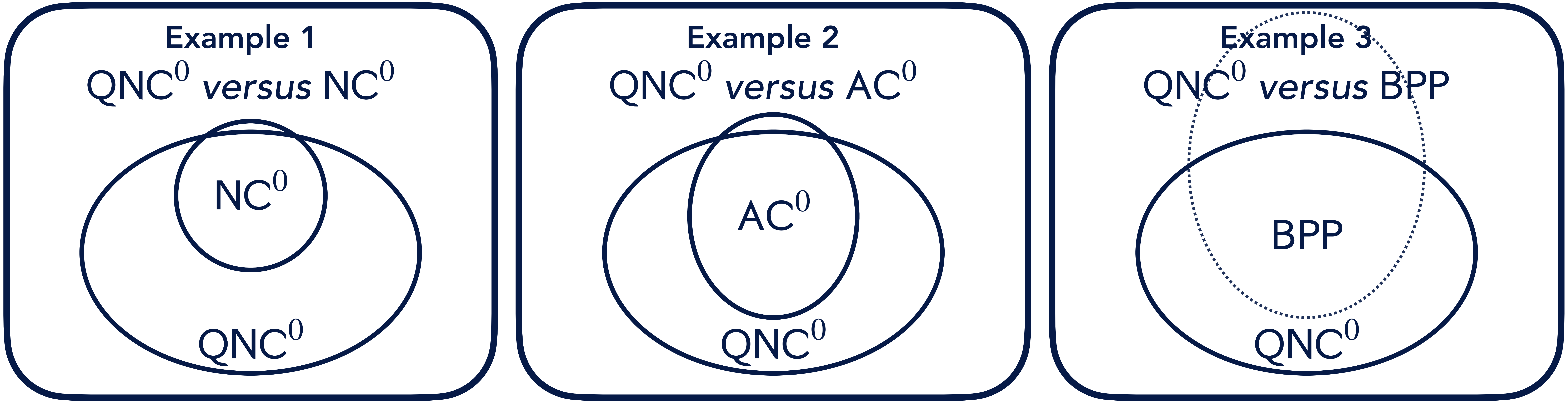


AC^0	Shallow circuits w/ unbounded fan-in AND gates

Exponential separation between shallow quantum circuits and unbounded fan-in shallow classical circuits. STOC 2019

Shallow quantum circuits

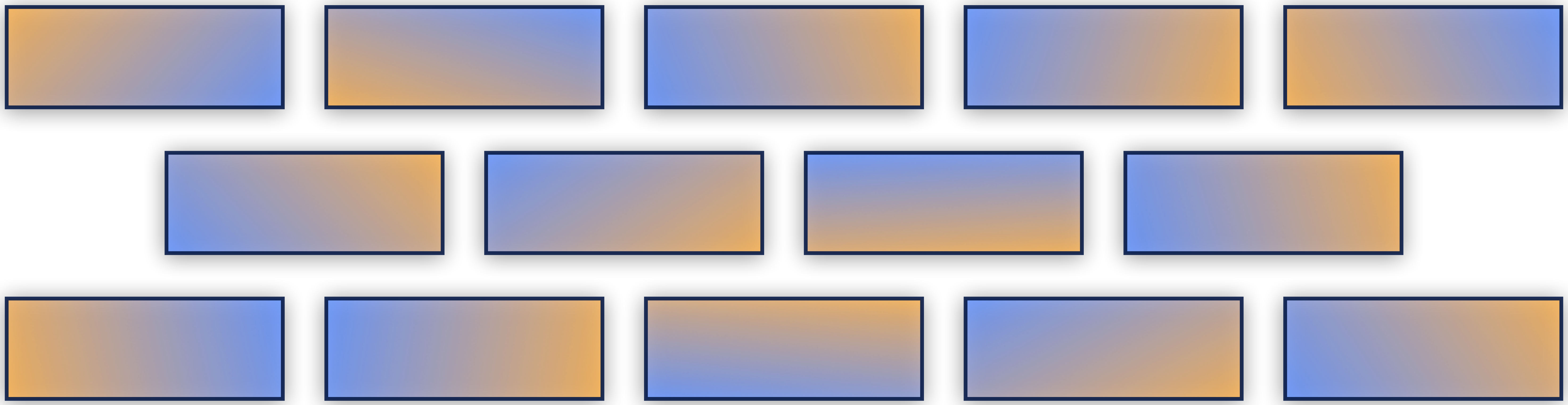
- **Example 3:** Shallow quantum circuits are **classically hard** to simulate (for sampling tasks) assuming PH does not collapse.



**Assuming PH does not collapse*

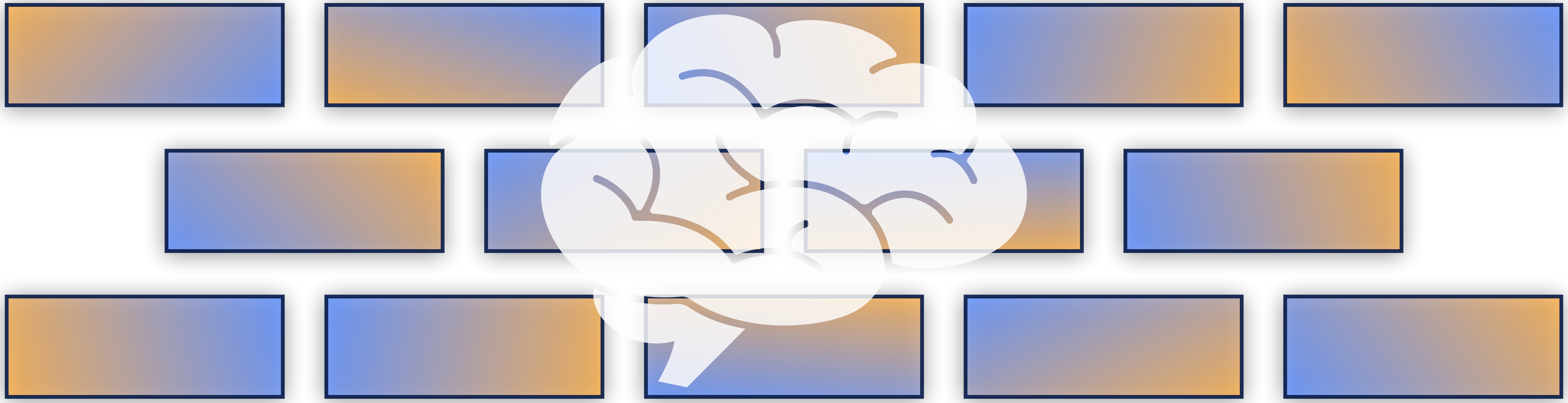
Learning shallow quantum circuits

- The power of QNC^0 raises the hope that they may be used as machine learning (ML) models with quantum advantages.



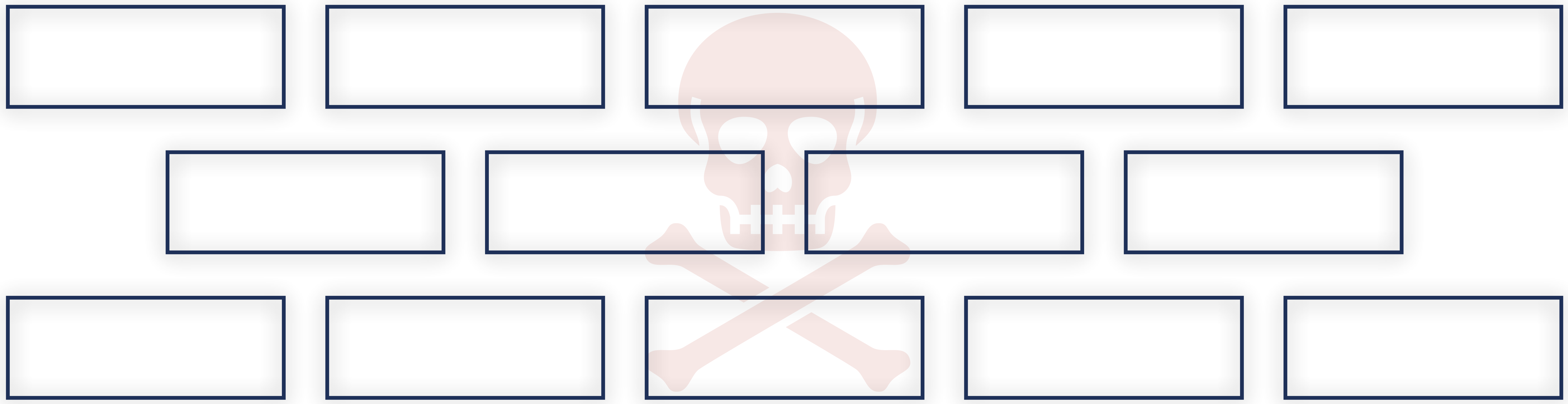
Learning shallow quantum circuits

- Extensive prior works have studied the problem of learning/training parameterized shallow quantum circuits.
- a.k.a. shallow quantum neural networks (**QNNs**).



Learning shallow quantum circuits

- Unfortunately, despite the significant interest in learning shallow quantum circuits, **no efficient learning algorithm** is known.



Overview

- Challenges in learning shallow quantum circuits
- Provably efficient learning algorithm
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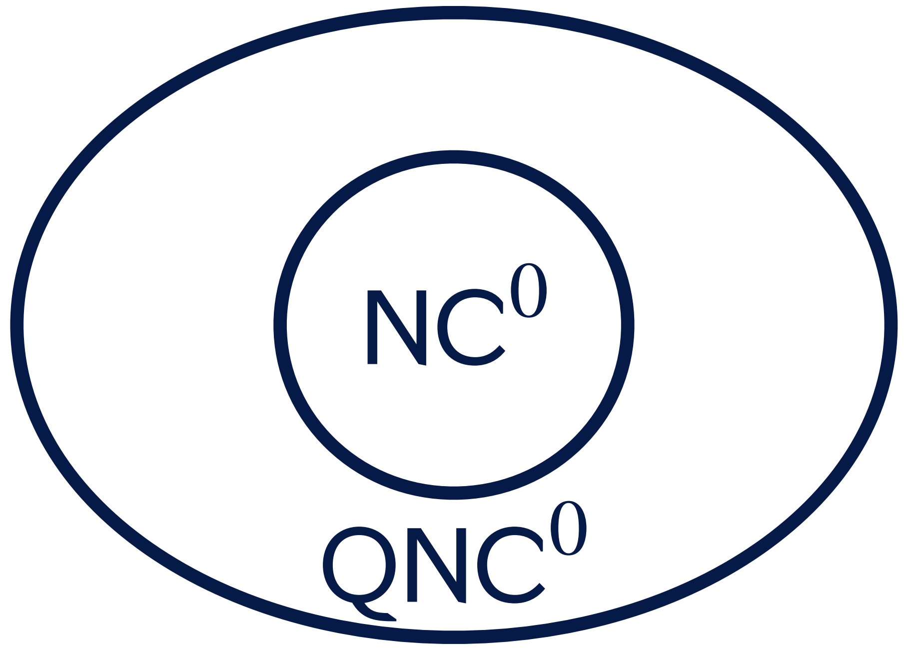
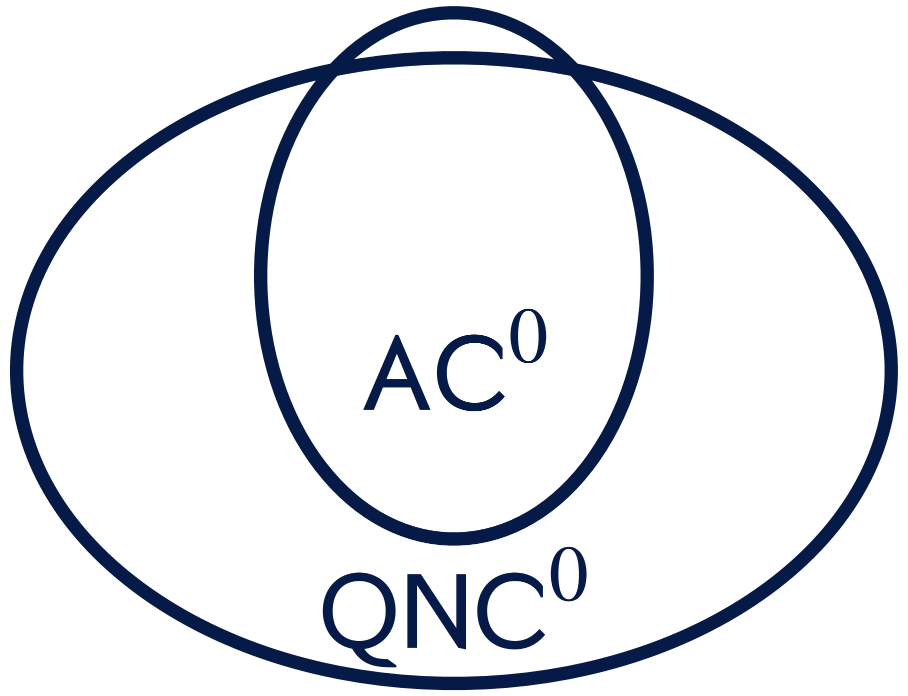
Challenge 1

- Shallow classical circuits are easy to learn:

1. NC^0 can be learned in poly. time
2. AC^0 can be learned in quasi-poly. time

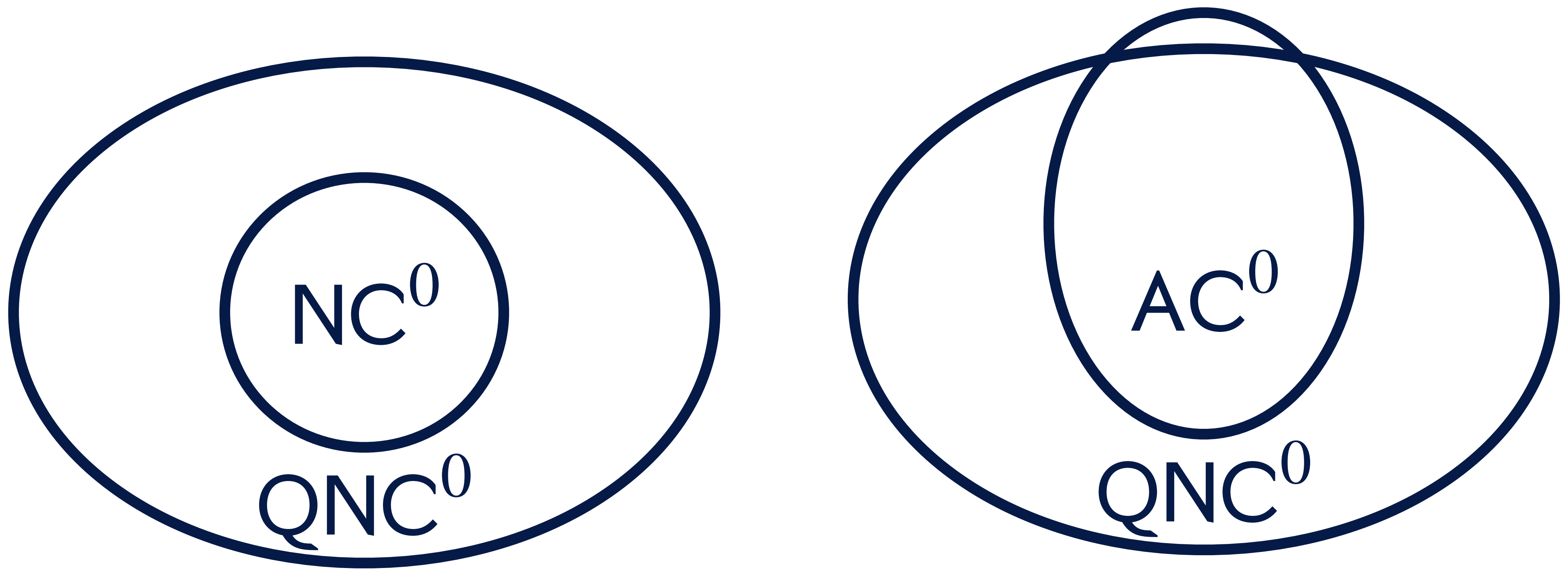




NC^0	Problems solvable by shallow classical circuits
AC^0	Shallow circuits w/ unbounded fan-in AND gates

Challenge 1

- But shallow quantum circuit can generate **classically-hard highly-nonlocal** correlations.
- Hence, known techniques do not apply.



NC^0	Problems solvable by shallow classical circuits
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Challenge 2

Shallow quantum circuits do not have barren plateau.
But they are swamped with **exponentially many bad local minima.**

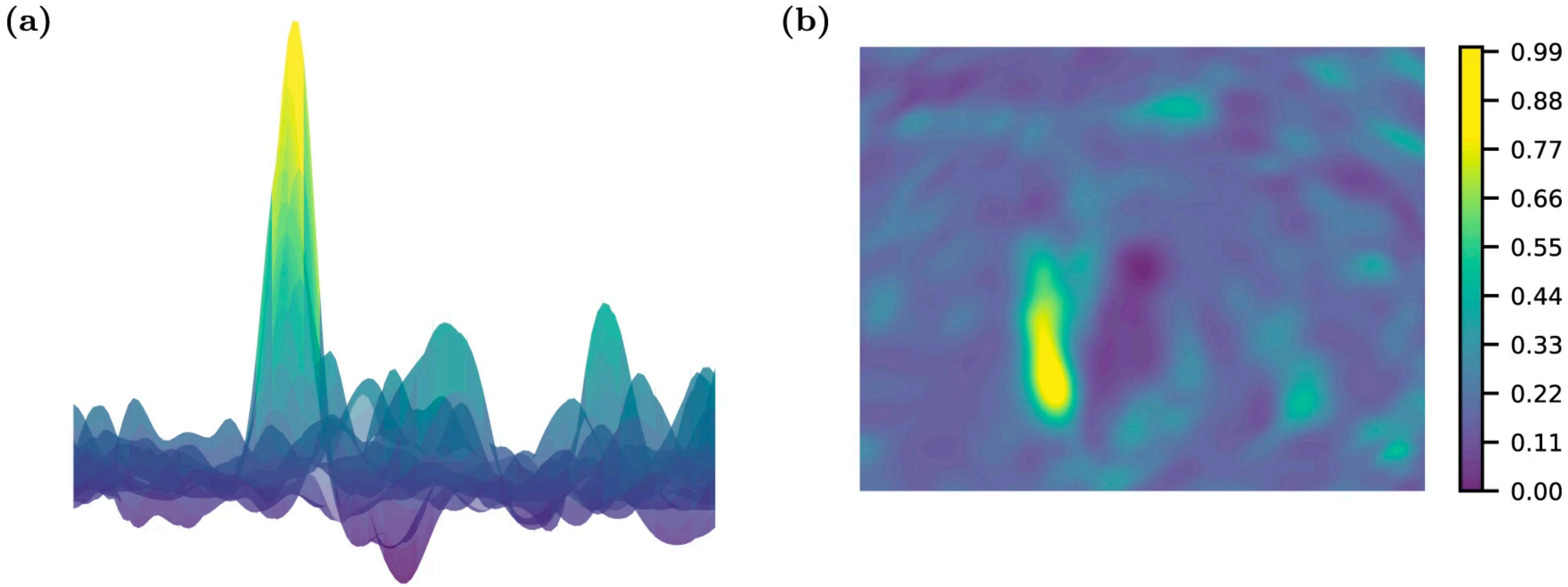
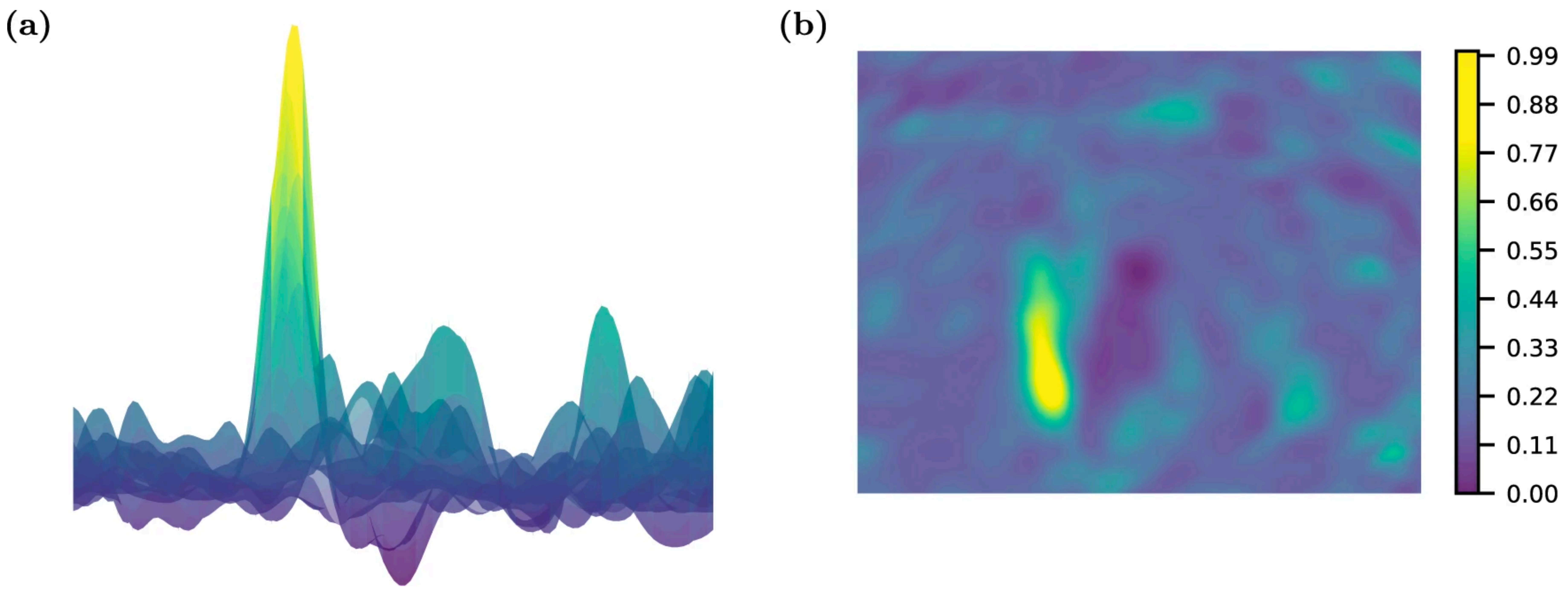


Fig. from "Quantum variational algorithms are swamped with traps". Nat. Comm. 2022

Challenge 2

As a result, gradient descent and other optimization methods **get stuck** in a bad local minimum and fail.



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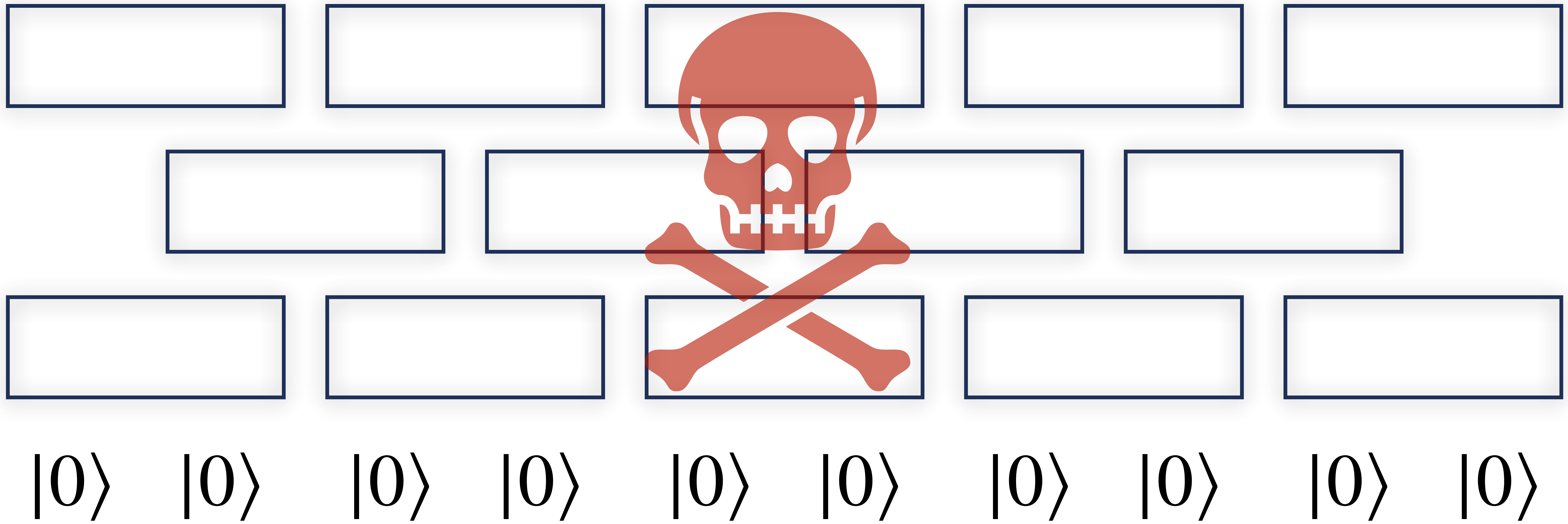
Question

Are shallow quantum circuits **computationally hard** to learn?



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Learning shallow quantum circuits

- Consider an unknown n-qubit shallow quantum circuit U.
- How to learn U efficiently?

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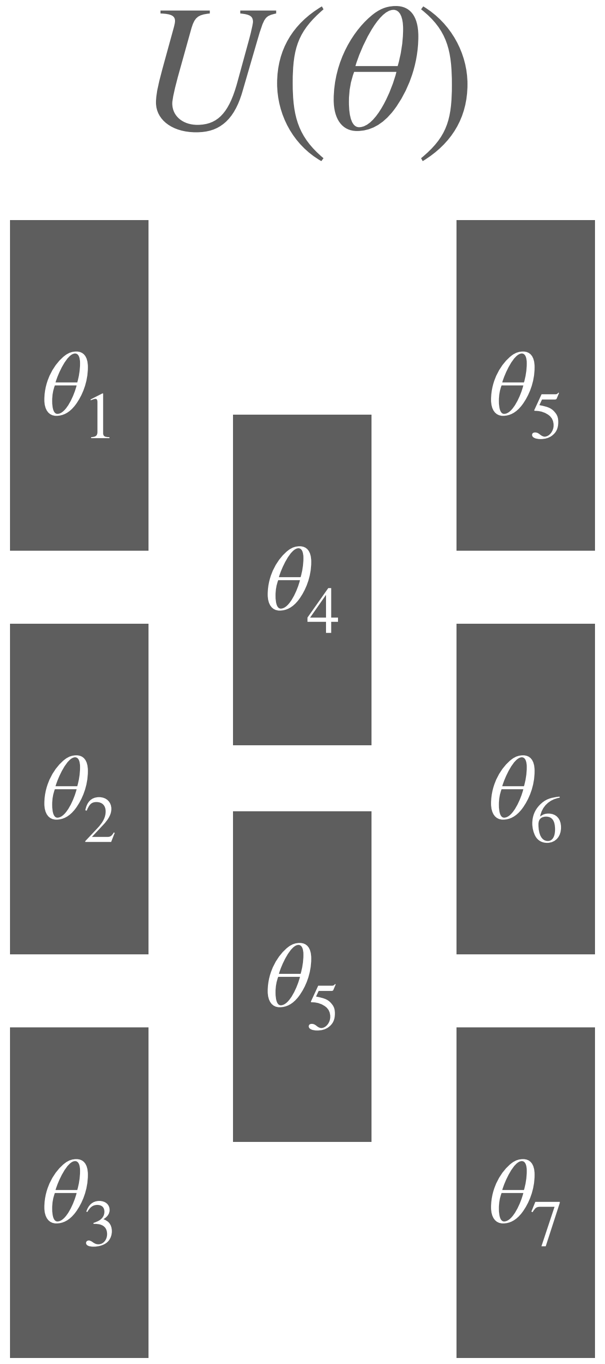
Standard strategy

1. Guess a param. circuit $\hat{U}(\theta)$.
2. Check the loss function.
3. Update θ, repeat 2.



Learning shallow quantum circuits

- Consider an unknown n-qubit shallow quantum circuit U.
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The expon. large search space has expon. many traps.

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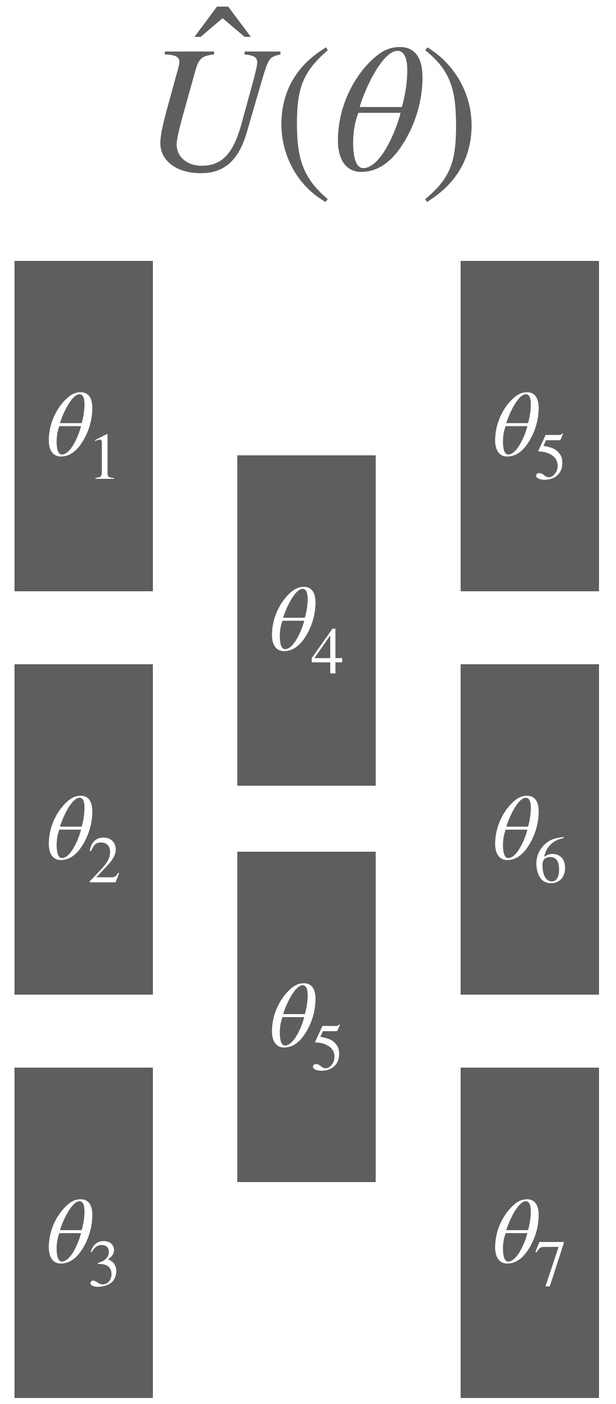


Learning shallow quantum circuits

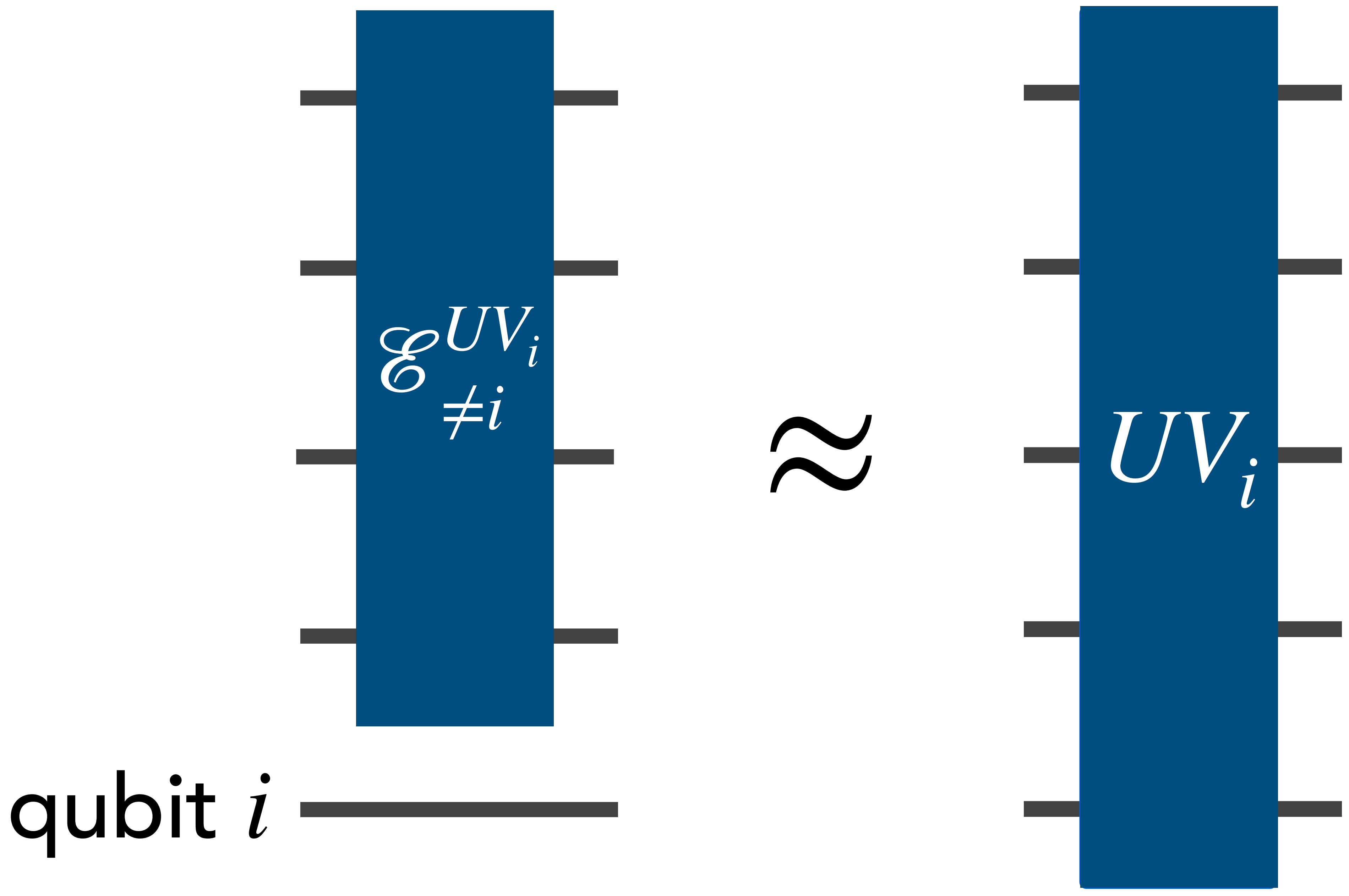
- Consider an unknown n-qubit shallow quantum circuit U.
- Use an unconventional parameterization of U.

Proposed strategy

1. Parameterize U by its local inversions.
2. Learn/train local inversions.

Local inversions

- We say V_i is the local inversion of U on qubit i if



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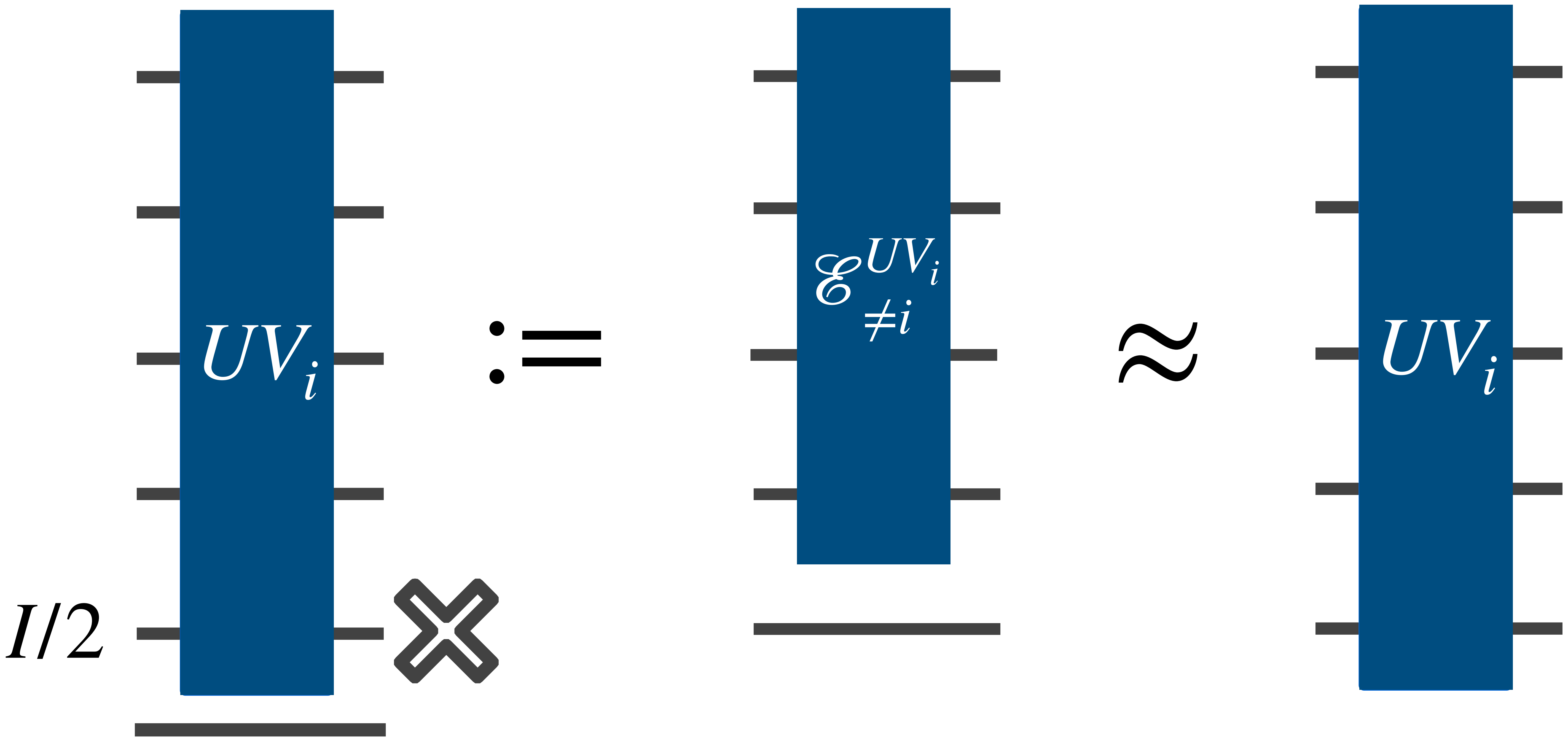

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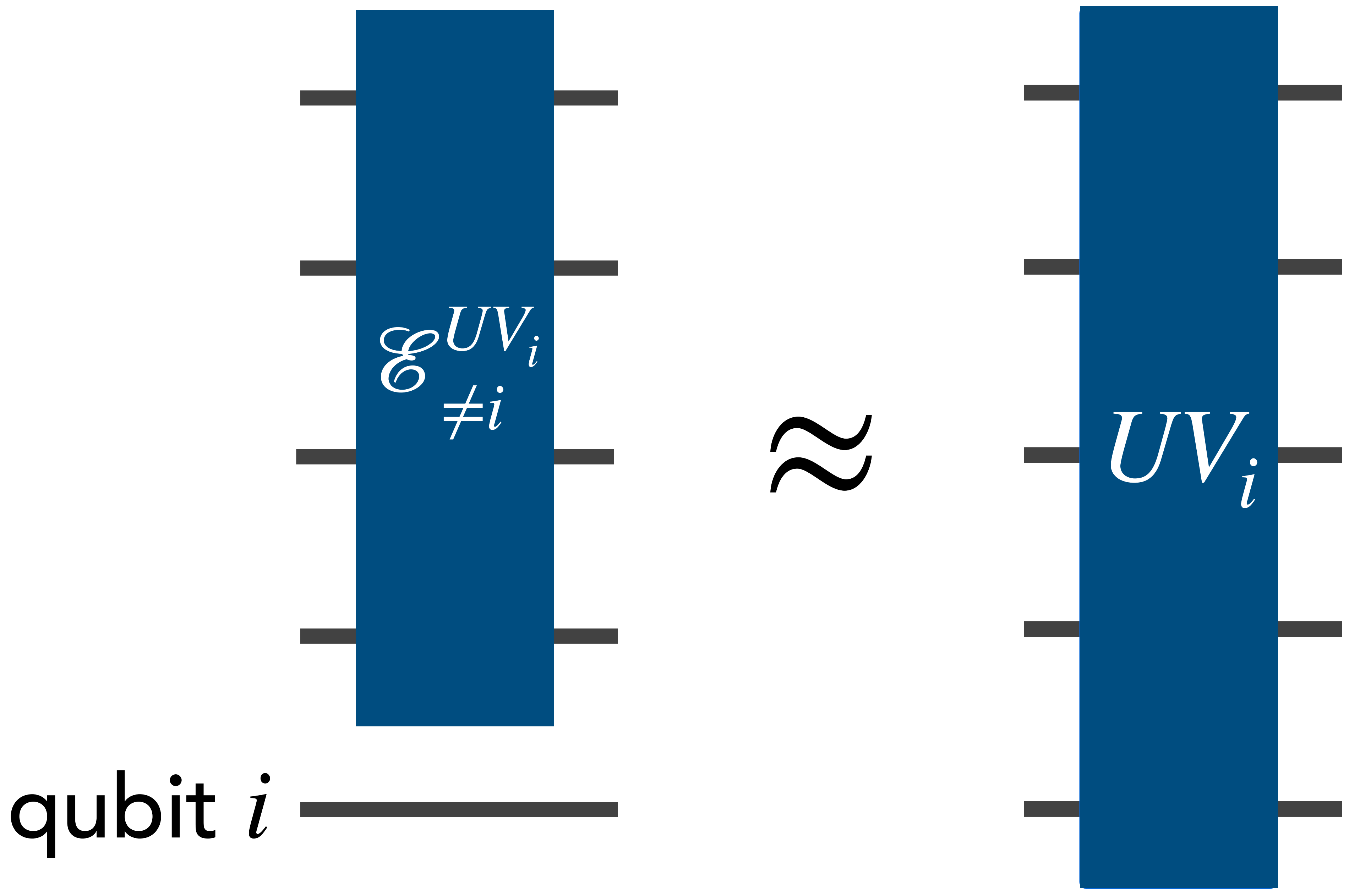
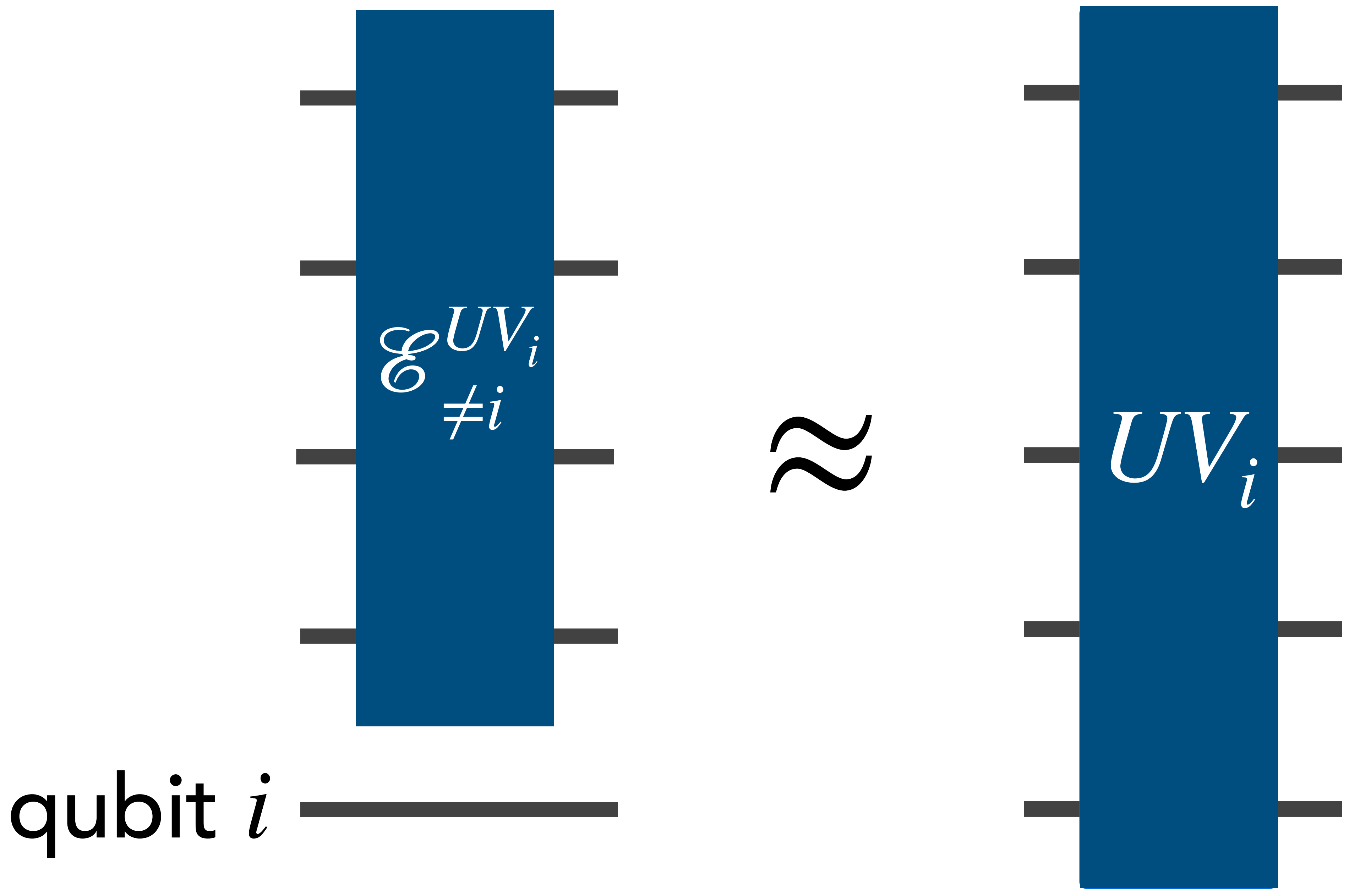


Local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- Does the local inversions uniquely parameterize U?



Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We can sew local inversions together to form U.

Initialize

$I/2 \quad I/2 \quad I/2 \quad I/2 \quad I/2 \quad I/2 \quad I/2$

$|\psi\rangle$

$2n$ qubits

Sewing local inversions

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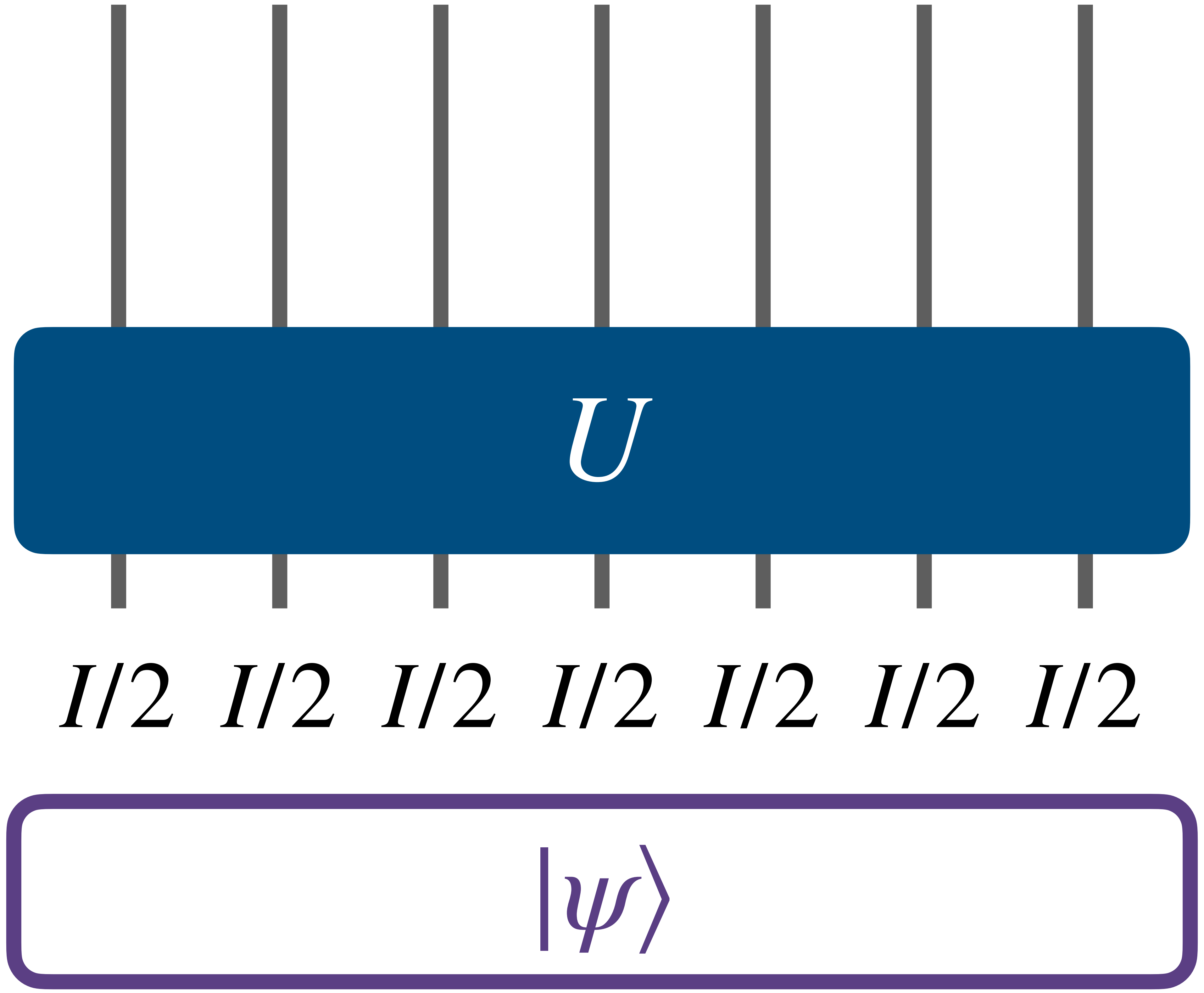
Imagine



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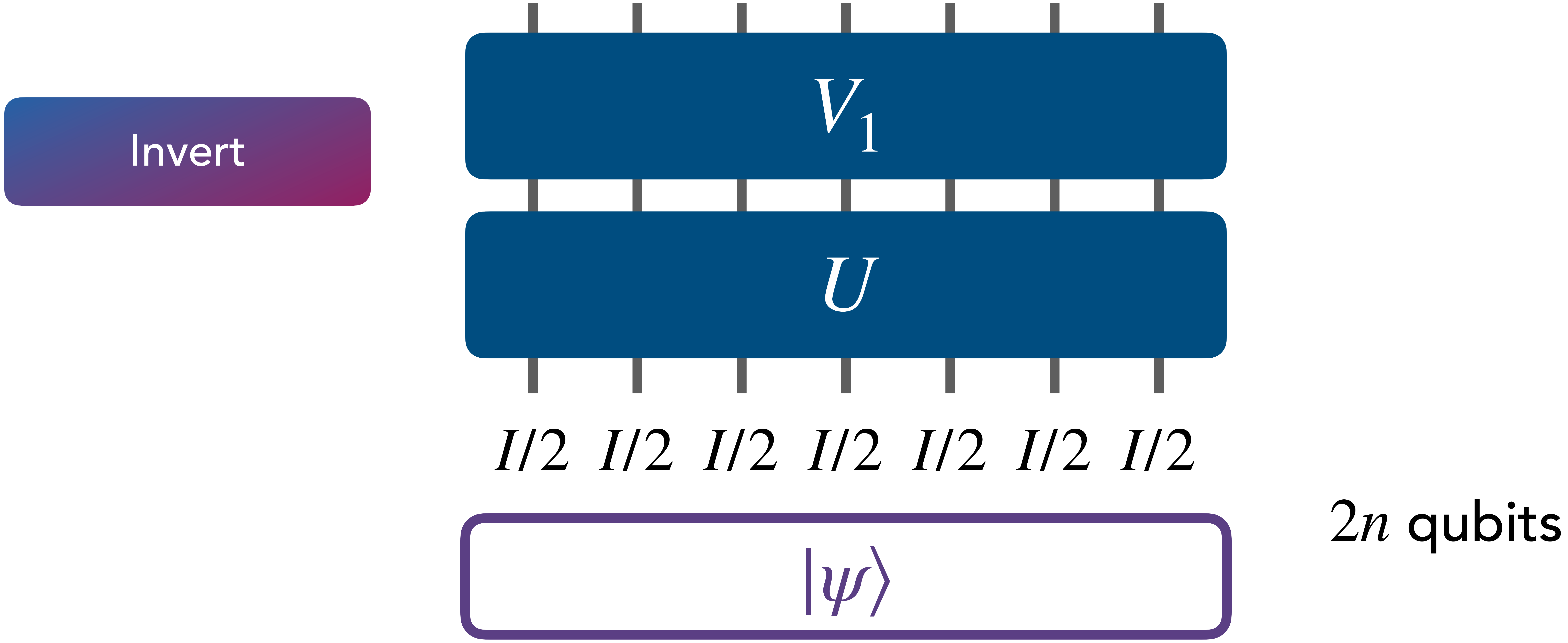

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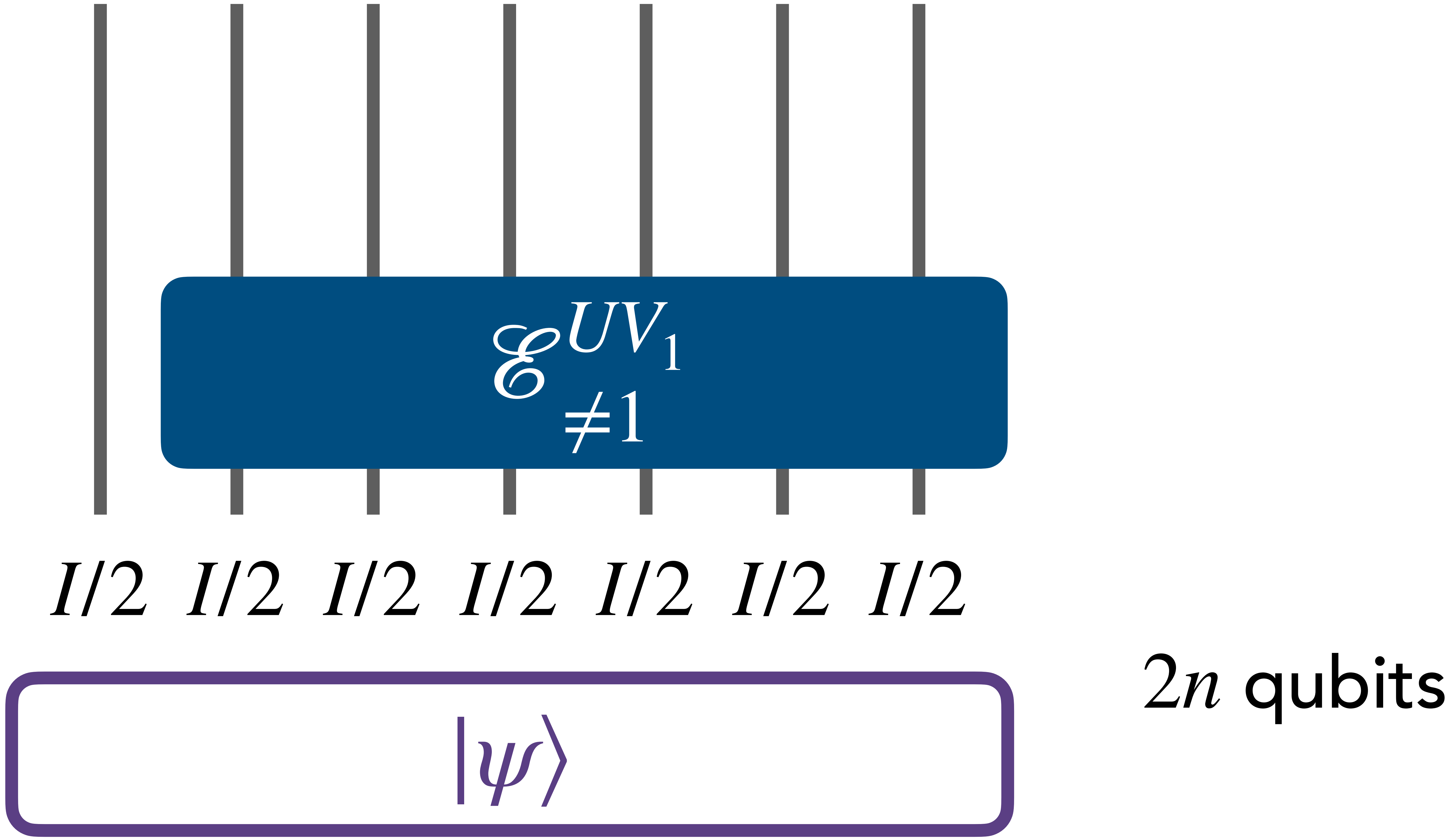
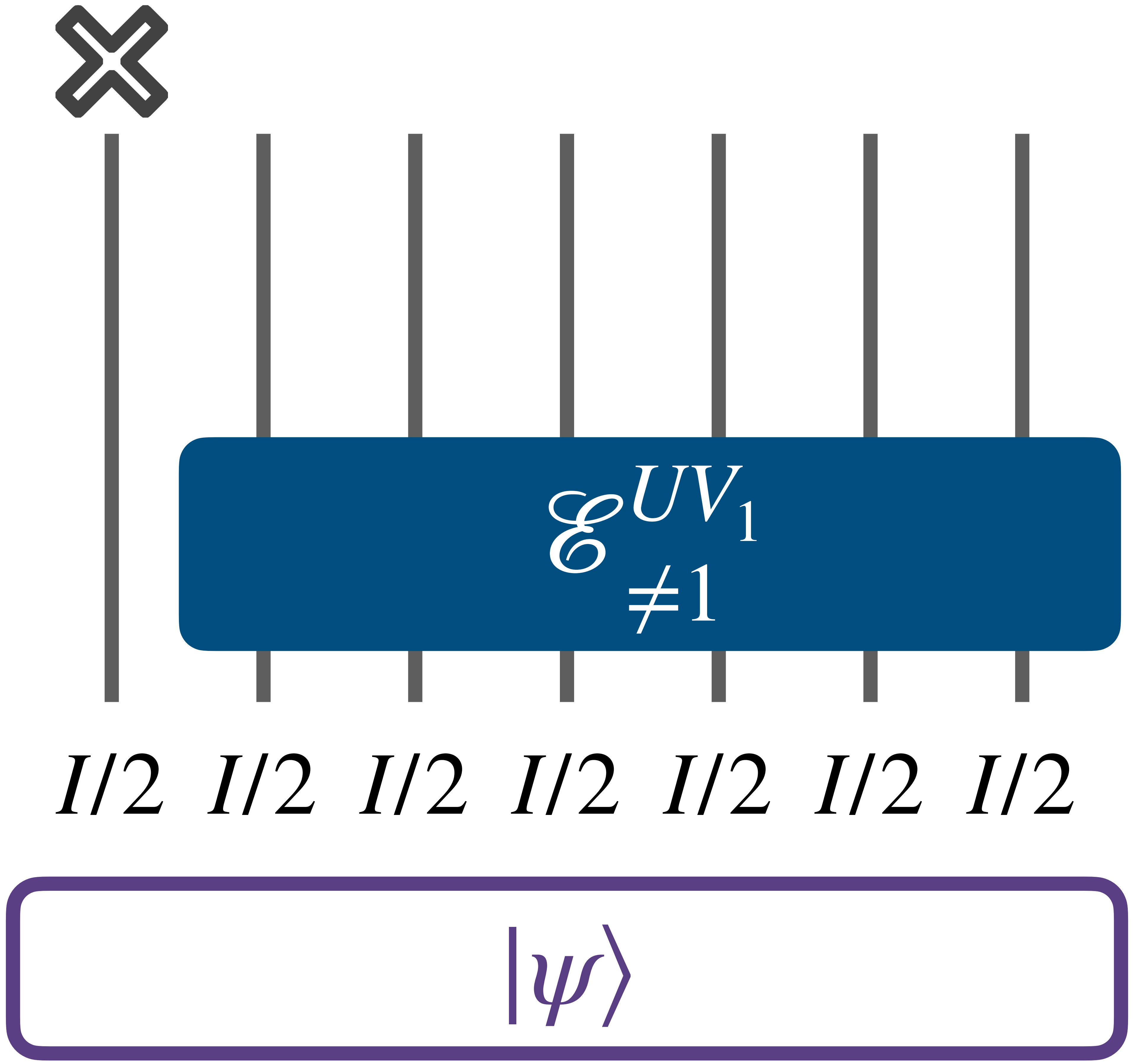
Uncover



Sewing local inversions

Dispose



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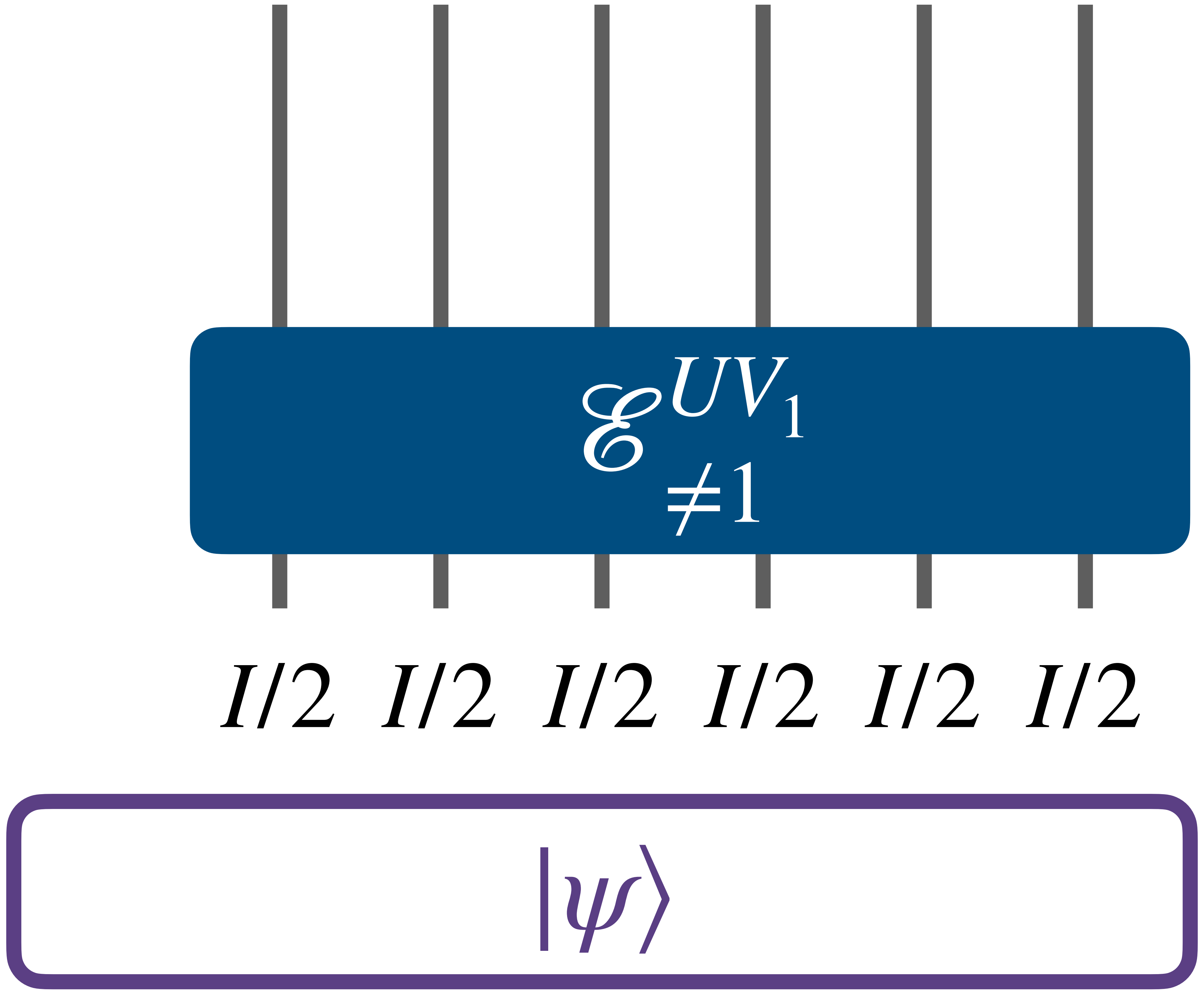

$2n - 1$ qubits

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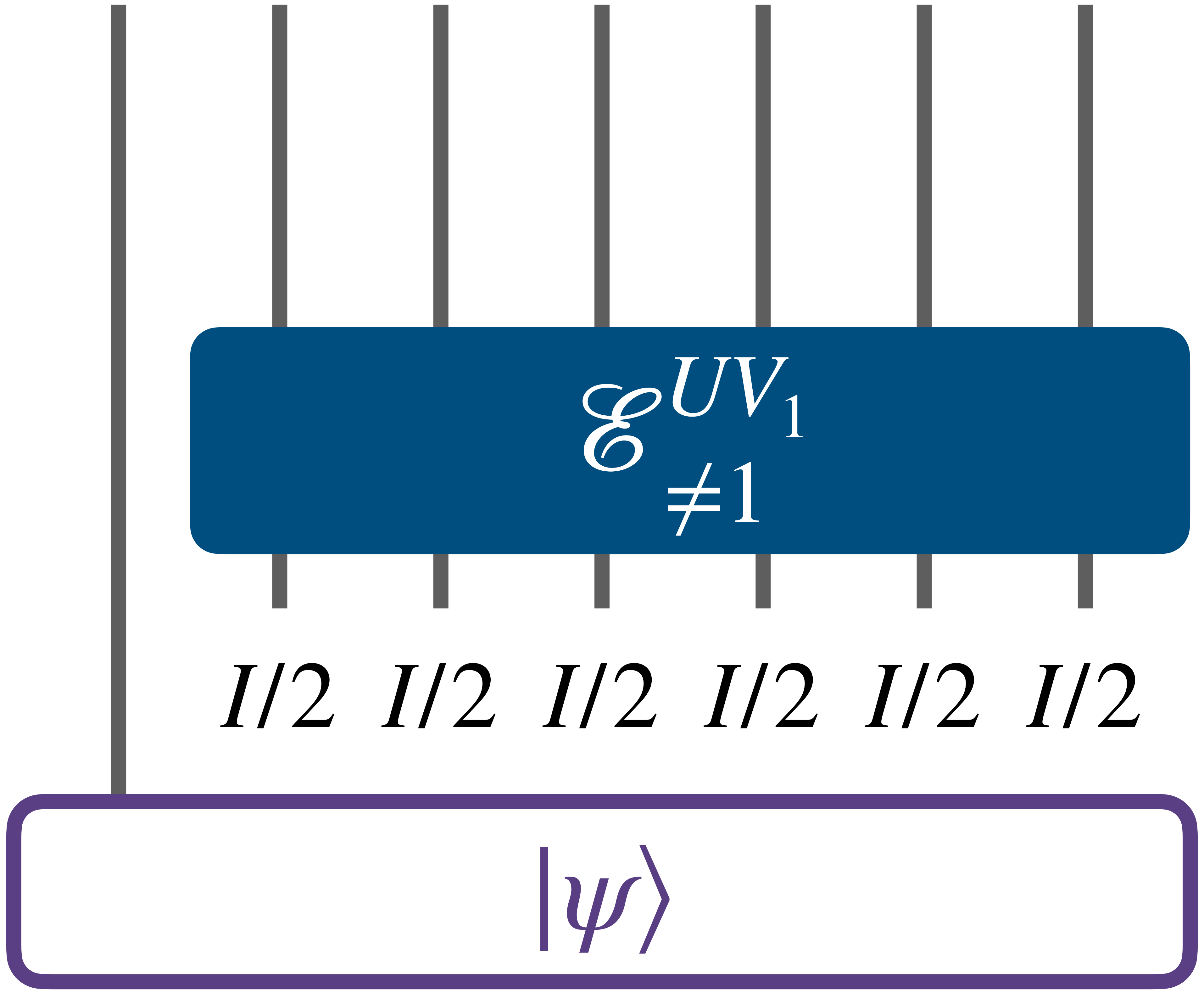
Rewire



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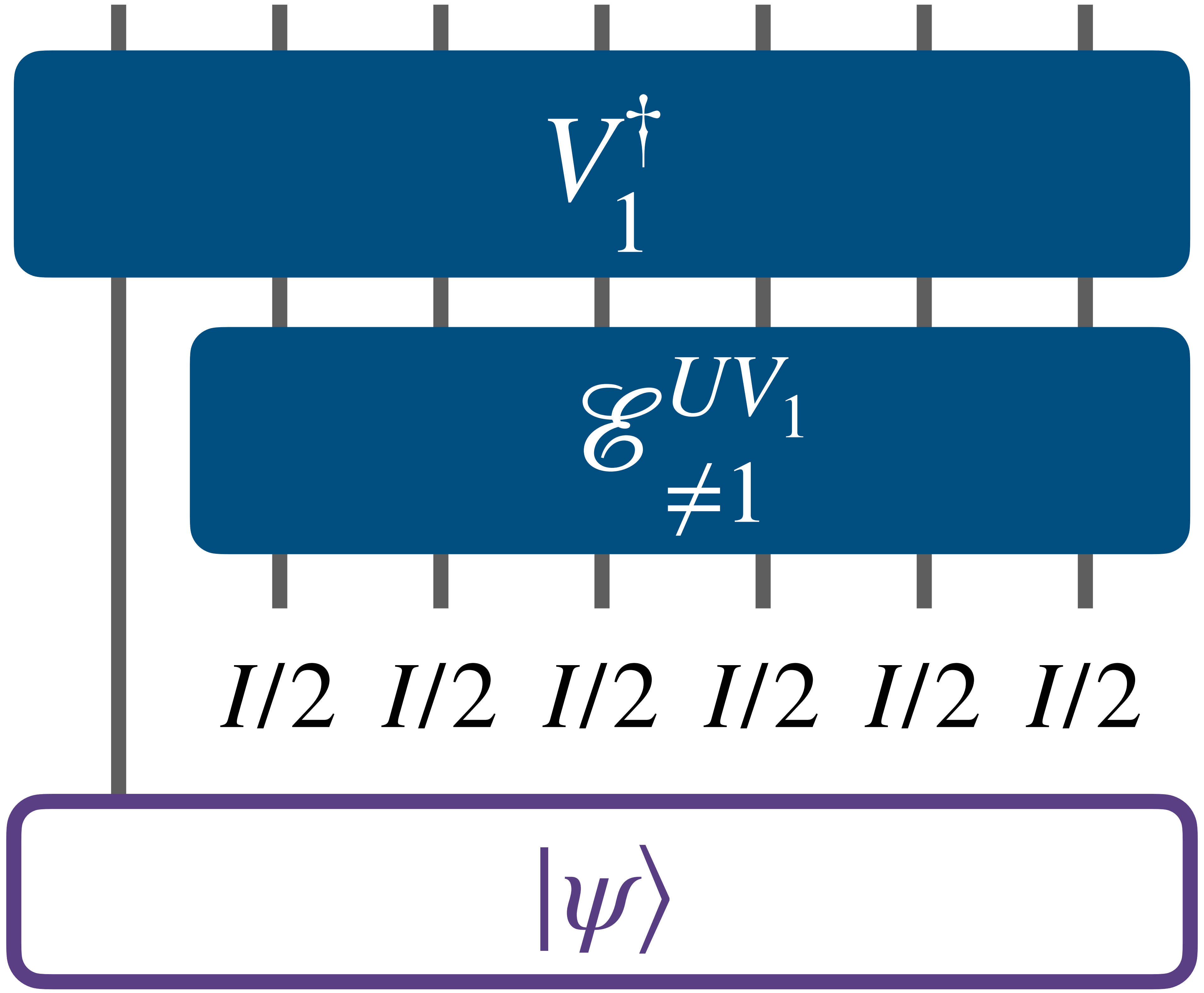
Un-invert



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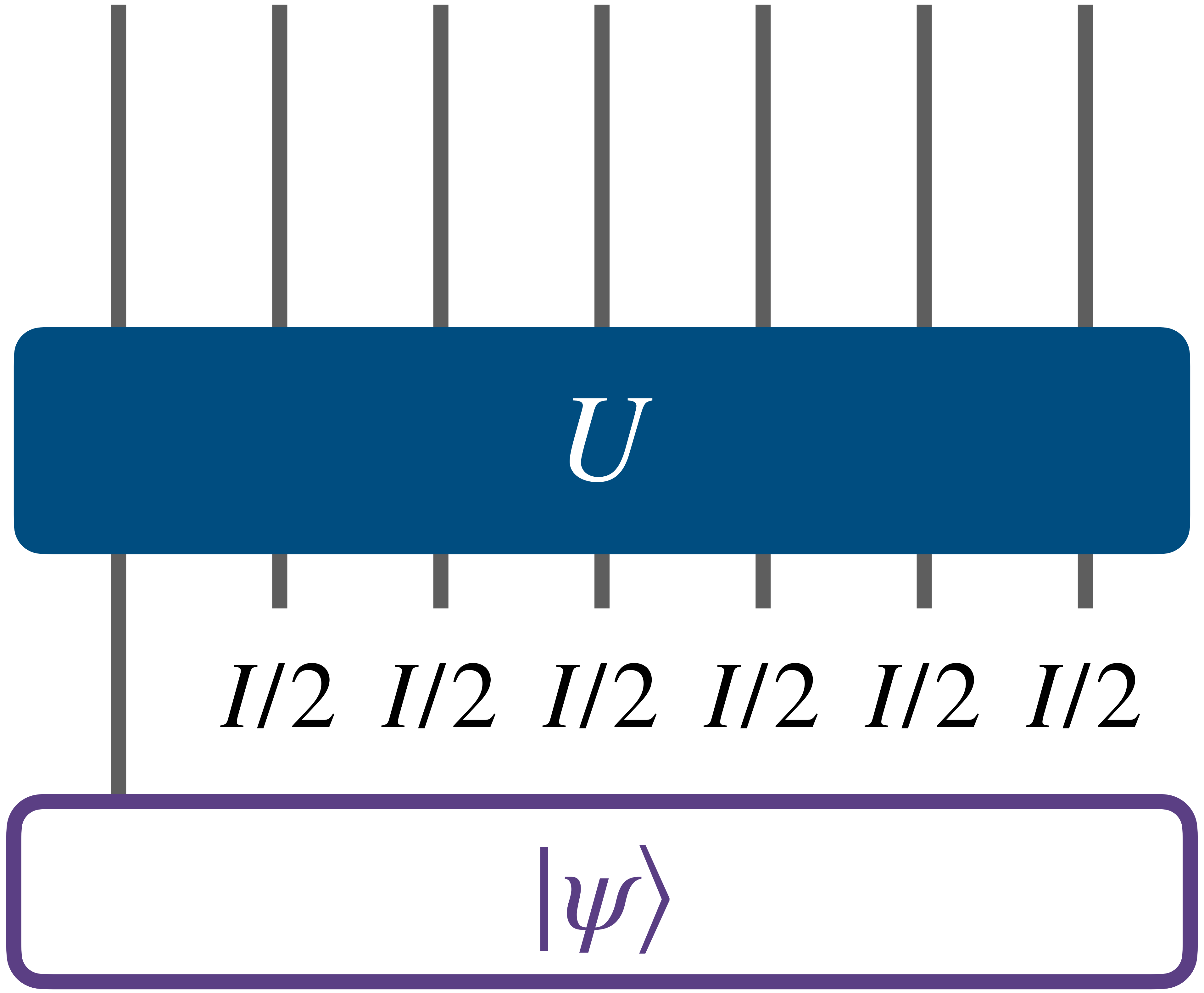
Heal



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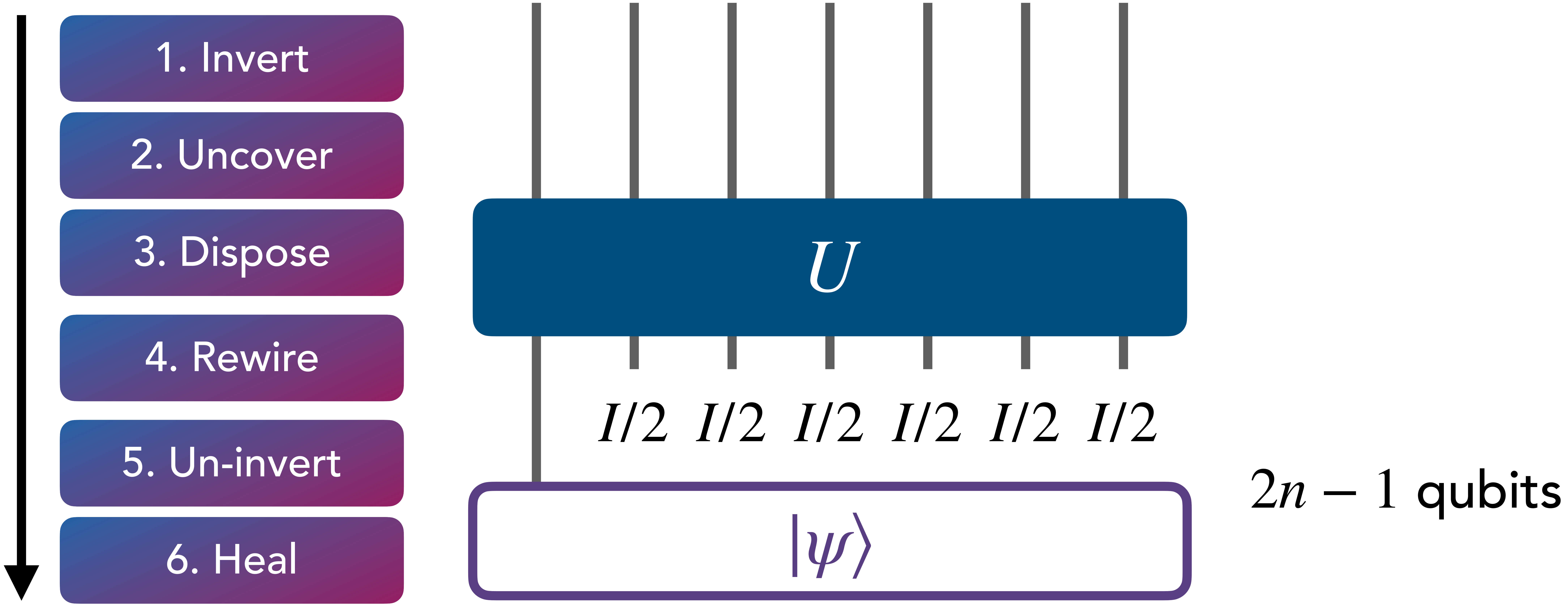

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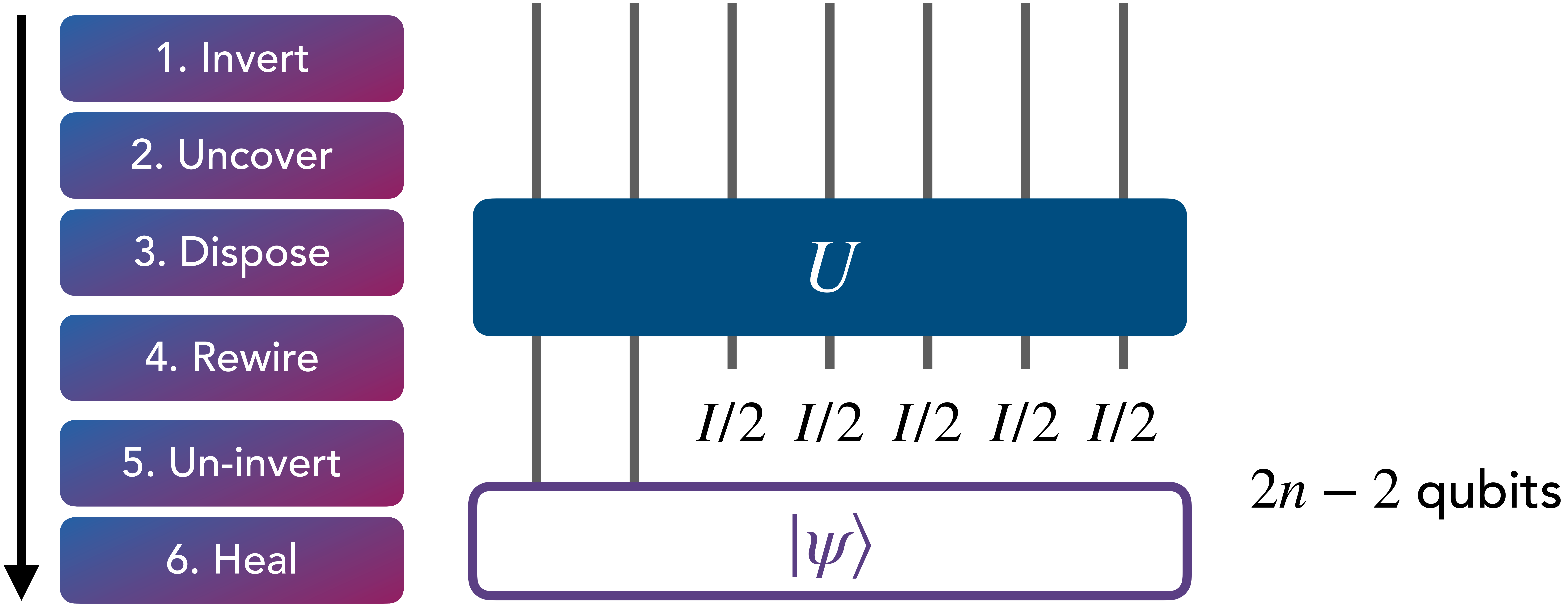

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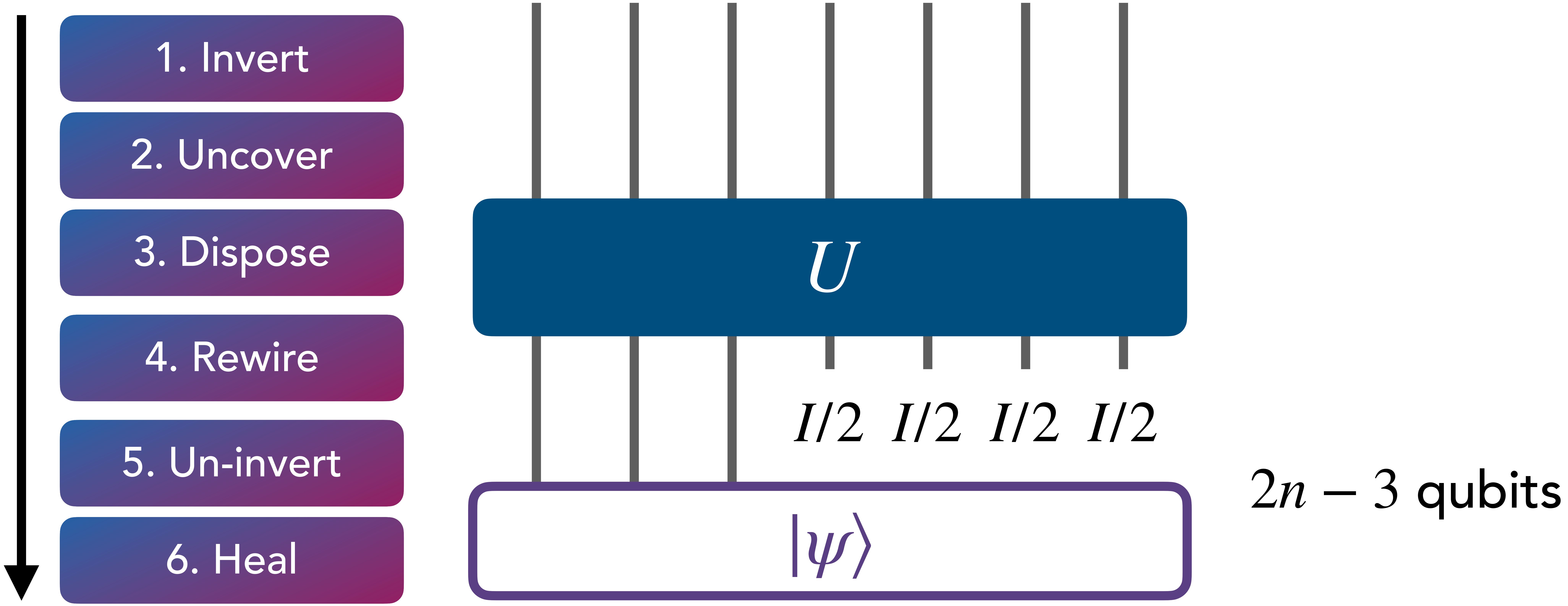
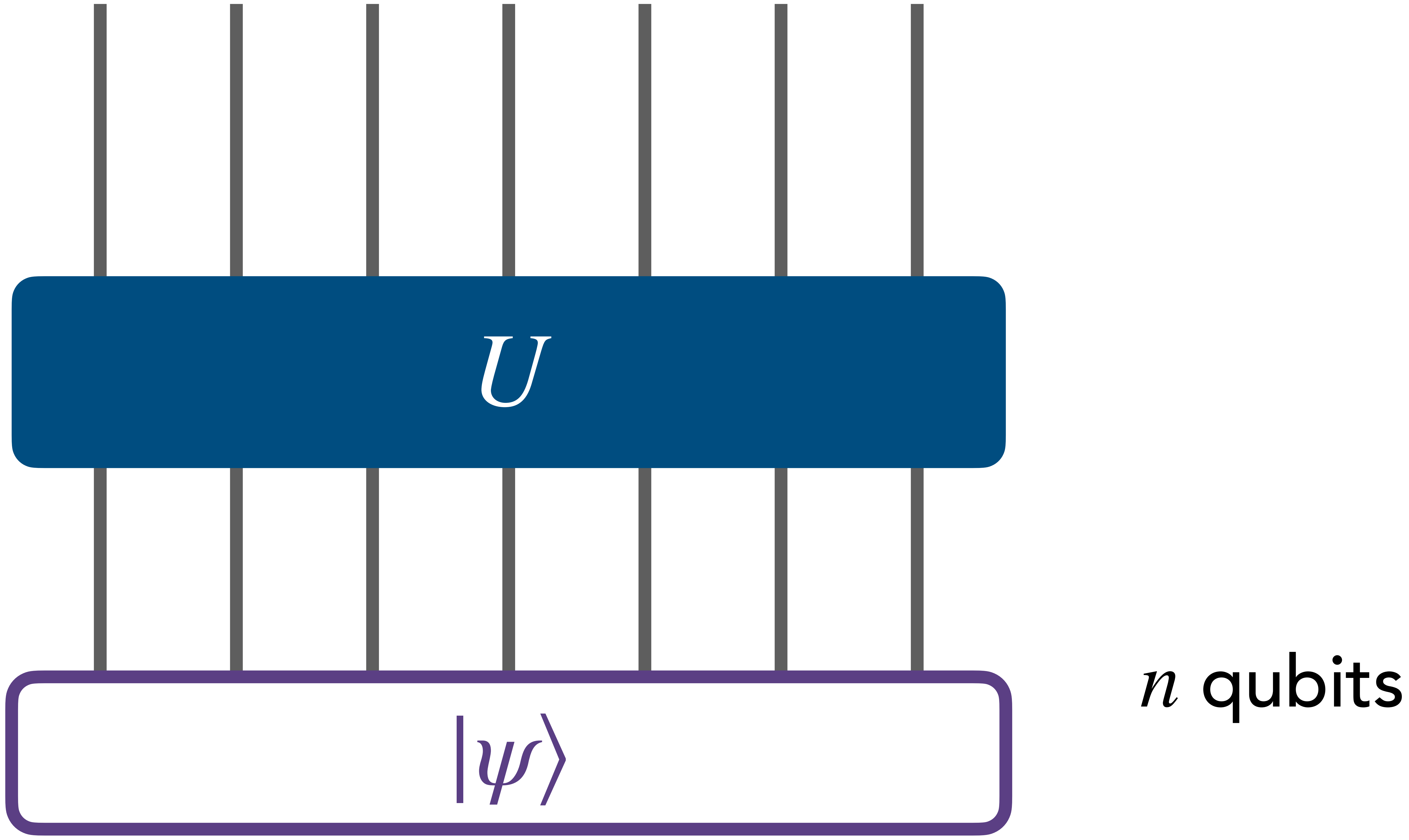


Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We have sewn together to form the n-qubit unitary U.

1. Invert
2. Uncover
3. Dispose
4. Rewire
5. Un-invert
6. Heal



Learning shallow quantum circuits

Theorem 1

Any n-qubit shallow quantum circuit U can be learned to ϵ diamond distance in $\mathrm{poly}(n, 1/\epsilon)$ time.

- The quantum circuit can be of any connectivity.

Learning shallow quantum circuits

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Any n-qubit shallow quantum circuit U can be learned to ϵ diamond distance in $\mathrm{poly}(n, 1/\epsilon)$ time.

- The algorithm only uses a classical dataset describing product state inputs to U and Pauli measurement outcomes.

Learning outputs of shallow quantum circuits

Theorem 2

Any state ρ prepared by n-qubit shallow 2D circuits can be learned to ϵ trace distance in $\mathrm{poly}(n, 1/\epsilon)$ time.

- The algorithm only uses a classical dataset describing Pauli measurement outcomes on the state ρ.

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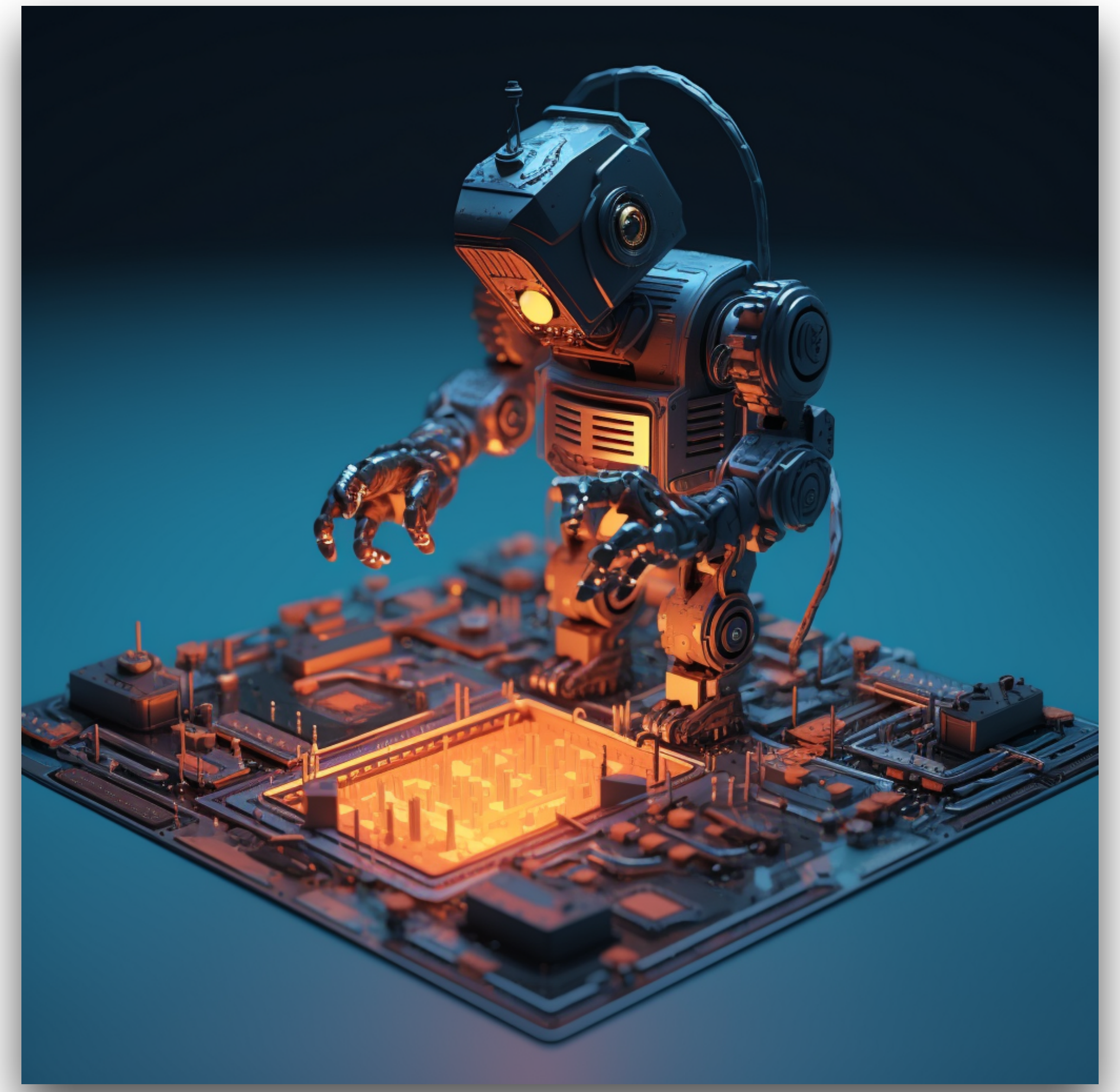
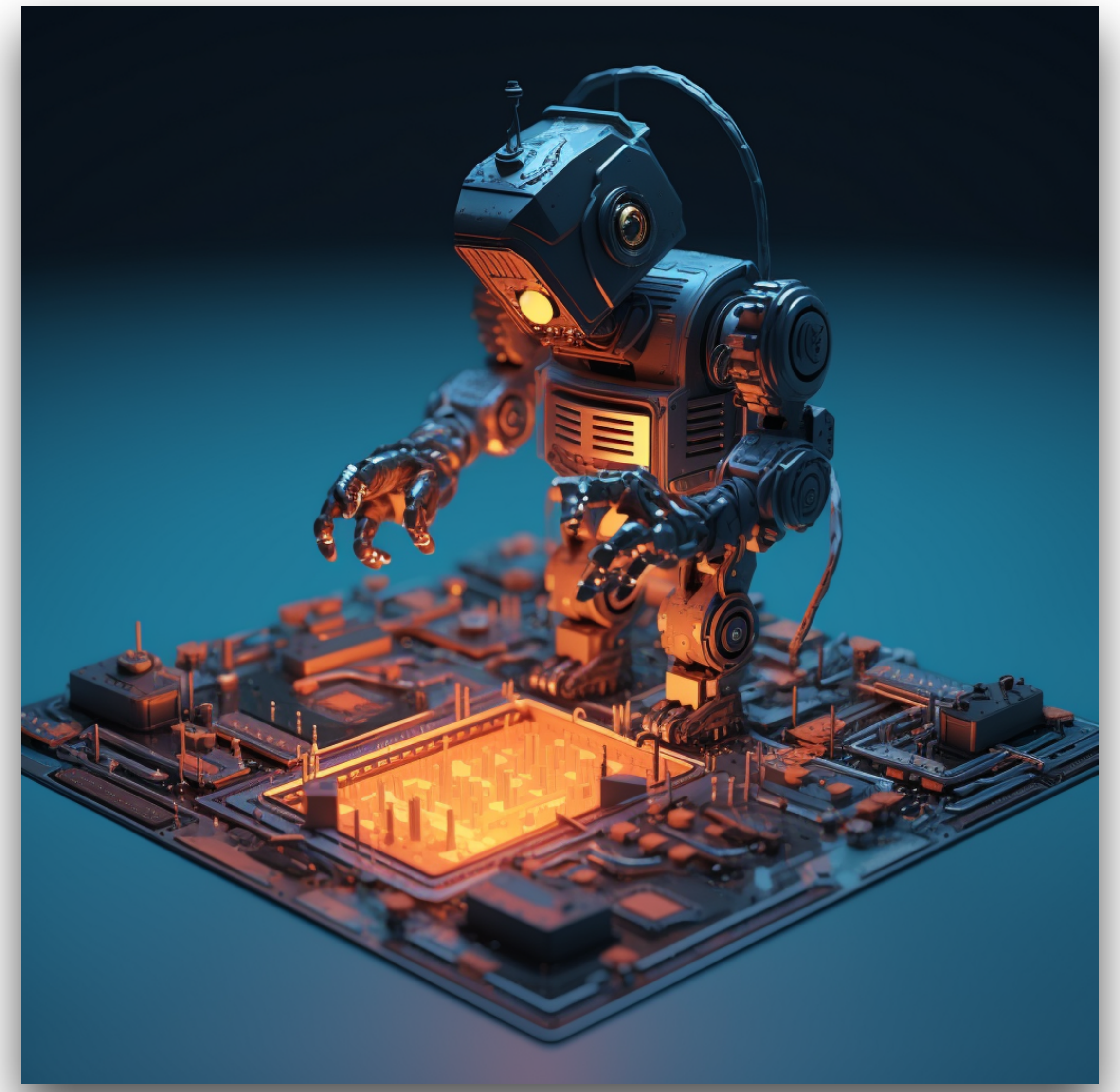
Applications



- What can we do with an efficient algorithm for learning shallow quantum circuits?

Applications



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Example 1

Learning quantum dynamics

Any unitary generated by short-time dynamics is learnable

$$X_i \to O_{X,i}$$
$$Y_i \to O_{Y,i}$$
$$Z_i \to O_{Z,i}$$

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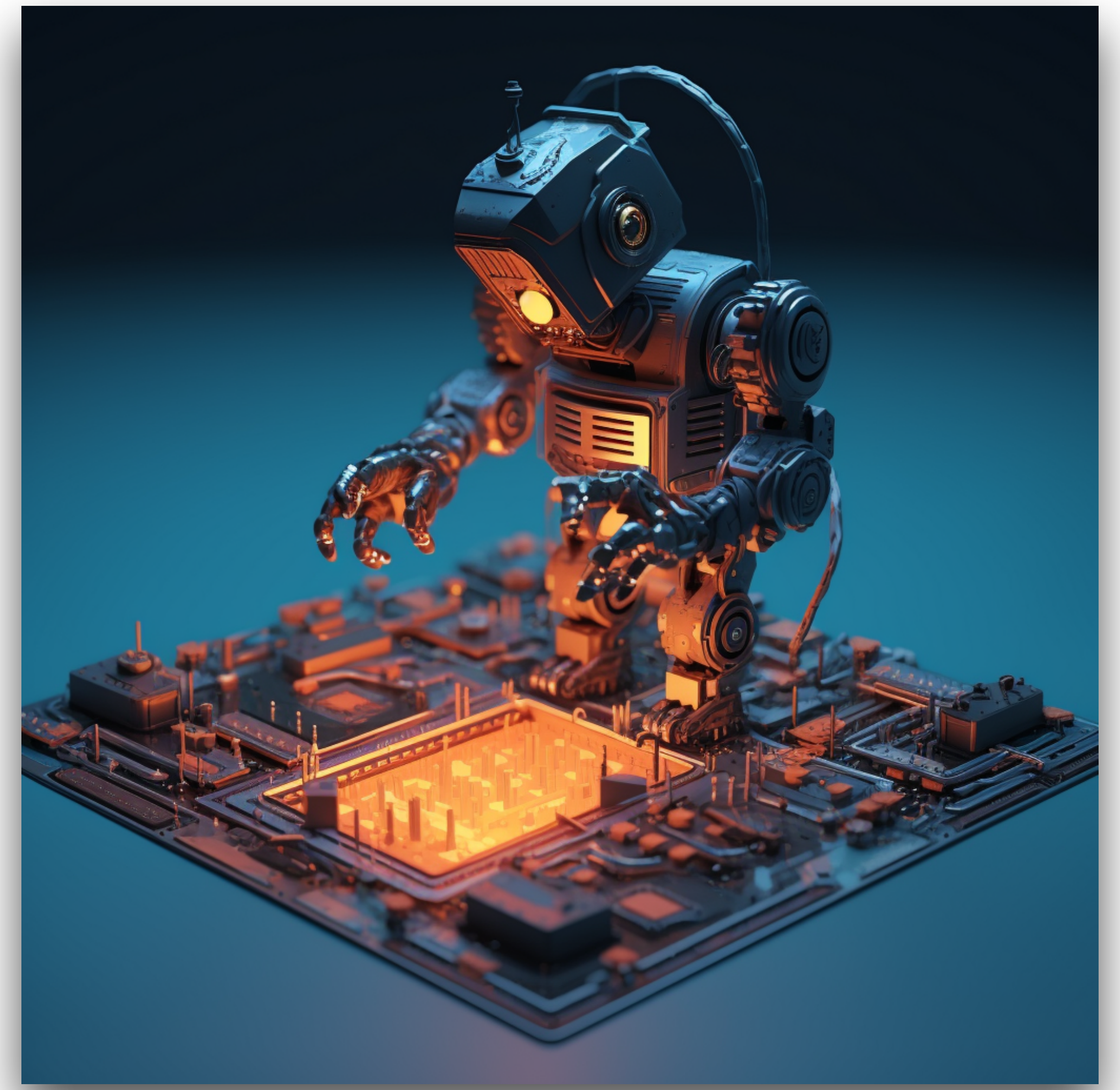

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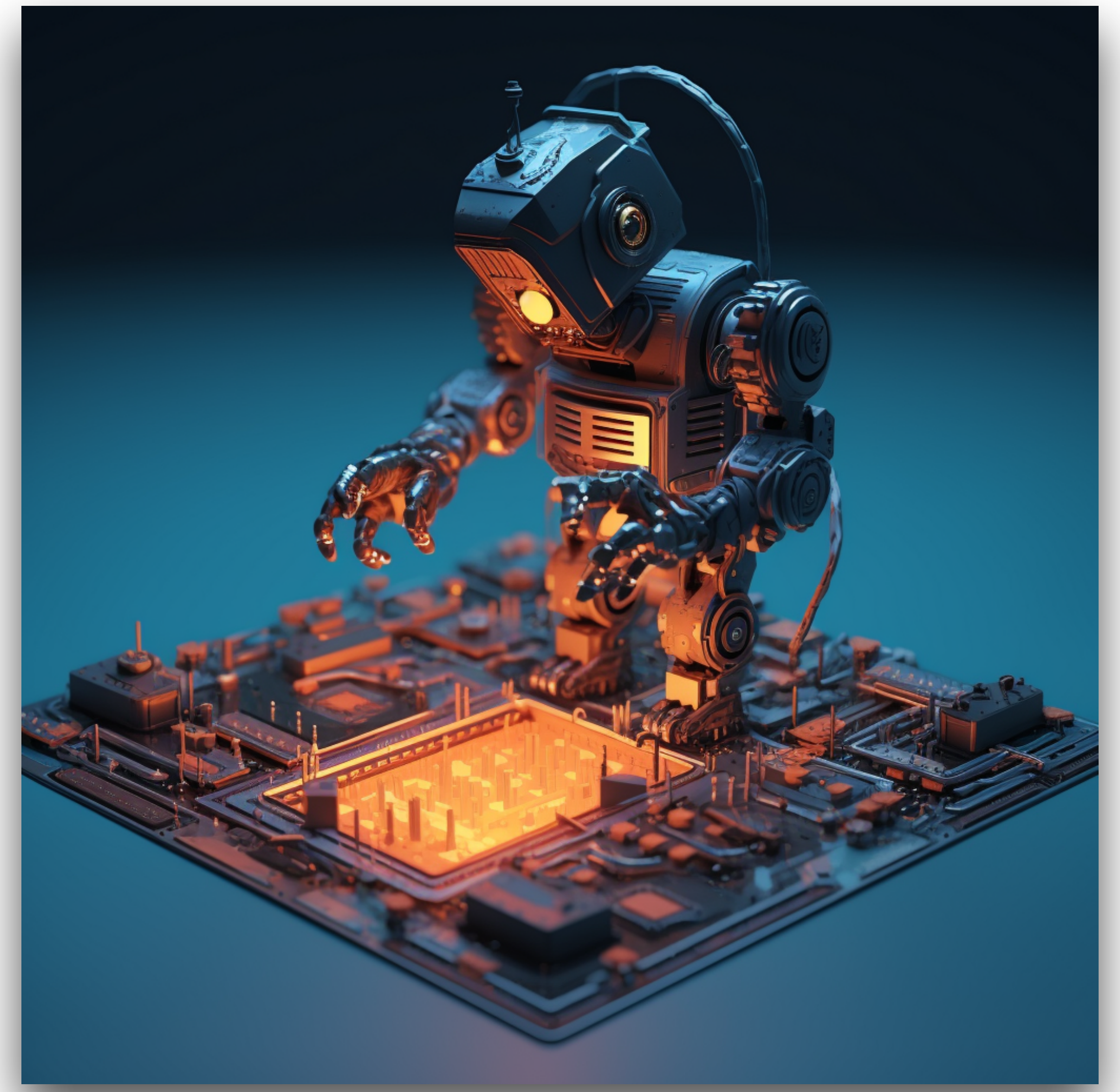
Example 2

Compressing circuits

Given an n-qubit circuit. Find a shallow circuit to implement it.

$$U = \prod_{i:m\leftarrow 1} U_i$$

Applications



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Hamiltonian simulations, distribution learning

• • •

Conclusion

- Shallow quantum circuits are efficiently learnable and are classically hard to simulate.
- What about learning certain deep quantum circuits? Or learning high-dimensional states?