

TOOLS OF THE TRADE

Learning quantum states from their classical shadows

In quantum mechanics, a quantum many-body system is represented by a large complex matrix whose size scales exponentially with the number of particles. This intrinsic exponential complexity empowers quantum technologies but, at the same time, it makes it practically impossible to completely characterize, or learn, a quantum many-body system even of moderate size (the current limit of quantum tomography being 40–50 qubits). This is an issue given that learning quantum systems is central to the development of quantum technologies.

To overcome the problem of exponential scaling, we proposed a new method known as classical shadow tomography. It allows the efficient learning of quantum systems from very few measurements. The idea is not to learn the complete representation of the system (the exponentially large matrix), but a model of the system that enables the accurate prediction of many properties. Each measurement of the quantum system causes the system to collapse, hence, naively, one would need at least order M measurements to measure M properties, that is at least one measurement per property. Using classical shadow tomography, from only order $\log(M)$ measurements, it is possible to construct a model that can accurately predict the M properties of the system.

Classical shadow tomography learns the quantum system by randomly evolving the system and following with a measurement that collapses the quantum wavefunction.


The classical descriptions of the random evolution and the measurement outcome are combined to form the classical shadow of the quantum state. The basic intuition of this method is the following. One can think of a quantum system as a high-dimensional object. Each random evolution followed by measurement can be seen as a projection of this high-dimensional quantum object on a random direction determined by the random evolution to a classical shadow that is easy to analyse classically. Using the classical shadow created along different random directions, one can paint a broad picture of the original quantum system. The classical shadows of the quantum state can be used to predict various properties, including local observables, quantum fidelities, entanglement entropy, and the expected value of the Hamiltonian. Although classical shadows can be used to predict various properties that cannot be simultaneously measured, there are fundamental limitations to any prediction procedure based on classical post-processing that forbids accurate prediction of certain classes of properties. Developing a better

understanding of the fundamental limitations is an ongoing research effort.

The original paper introducing classical shadow

tomography considered multi-qubit systems and used a random evolution class known as Clifford unitaries ([open-source code is available](#)). However, the method is not limited to qubit systems and Clifford evolutions. In principle it is possible to construct classical shadows for any quantum system, including those studied in chemistry, material sciences, and solid-state physics. Recent works have shown that classical shadow tomography can be applied to fermionic systems and analogue quantum simulators based on fermionic Gaussian unitaries and chaotic Hamiltonian evolutions. The study of classical shadows for other classes of physical systems and other kinds of random quantum dynamics is a work in progress.

To conclude, classical shadow tomography is a general procedure for converting a quantum system into a classical representation. The method opens up all kinds of classical approaches for studying quantum systems. For example, one can combine classical machine learning models with classical shadows to rigorously predict ground states in quantum systems and classify exotic quantum phases of matter. I am hopeful that the combination of classical shadows and existing computational tools will contribute to the development of quantum technologies and physical sciences.

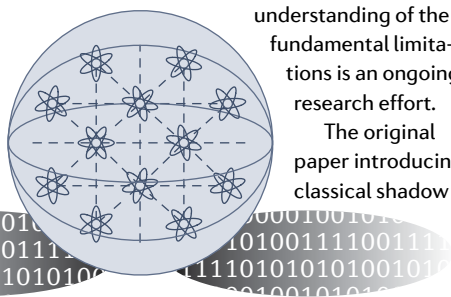
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Competing interests

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